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PART I

MECHANICS, SOUND, AND HEAT

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COLLEGE PHYSICS

MECHANICS

INTRODUCTORY

CHAPTER I

FUNDAMENTAL PRINCIPLES

1. Science and Natural Law. Physical Science is concerned with the discovery, investigation, description and explanation of phenomena in the inorganic, or inanimate, world.

The natural tendency of the human mind is to try to arrange the facts of daily observation according to some rational plan; to subject them to some general rule; in short, to explain them. A new fact is considered as explained when it has been shown to be in accord with previous knowledge and to conform to some more comprehensive statement of relationship. Thus daily experience shows that all bodies, such as wood, stone, lead, water, etc., if unsupported, fall to the ground, or if supported, they press upon the support; in other words, they have weight. Torricelli recognized as the cause of the pressure of the air the fact, already known to Galileo, that even gases have weight, and he showed that the ocean of air presses upon the earth's surface because of its weight. In this way the phenomenon of atmospheric pressure was brought into harmony with the facts of previous knowledge, and with the general proposition that all bodies have weight.

Such a proposition is called a law of nature. It has been found to be true in all cases observed; and while it may hereafter be included in some more general proposition, it can never be shown to be false. Science has been defined as "a body of generalizations so irrefragably true, that while they may be subsequently included in some larger generalization, they can never be overthrown." The law of weight has since been included in Newton's law of gravitation, but it has lost none of its truth thereby.

Physical Science embraces the related branches, Physics and Chemistry. The boundaries of these sciences are separated by no sharp line of demarcation, but overlap in many cases, and many laws are common to both. Their methods of attack are daily becoming more similar as their intimate relation is better understood. Their ultimate problem is the investigation of phenomena and the enunciation of laws pertaining to the constitution of matter and its relation to energy.

2. Matter. Matter may be defined as that which we can perceive by our sense of touch. A mass is a definite quantity of matter. Chemistry is occupied with the investigation of changes in the composition of matter. Its fundamental proposition is that of the "Conservation of mass." In accordance with this principle, it is asserted that the quantity of matter in the universe is constant, and that by no human agency can matter be created or destroyed.

Physics is concerned with matter only in so far as it serves as a *carrier of energy*. The fundamental proposition of Physics is the "Conservation of energy." This proposition asserts that the quantity of energy in the universe is constant; that energy, like matter, is indestructible; and that although it may be transformed and transferred in an endless round of changes, no energy is ever lost—the amount of energy remains the same. This does not mean that all the energy is available, or that it will remain so. Much of the energy at our disposal is wasted, in that it escapes in the form of uniformly diffused heat and is thereby rendered unavailable. It has not, however, been destroyed.

3. Inertia. Of the various properties of matter, such as extension, impenetrability, divisibility, porosity, compressibility, elasticity, weight and inertia, perhaps the most characteristic is that of inertia. Inertia is the resistance which matter offers to any change in its condition of rest or motion. Matter is powerless of itself either to move or to stop moving if once set in motion; moreover, it resists any attempt to move it if at rest, or to stop it if in motion.

Illustrations of inertia are seen in the hammering of the water in a water pipe on suddenly closing the faucet, in the action of the hydraulic ram or of the fly wheel of an engine. Familiar examples are also found in the stamping of snow from the feet, in the beating of dust from a carpet, in the motion of a bicycle rider when his wheel strikes a stone or in the case of a person who steps from a rapidly moving car while facing to the rear. More remarkable illustrations of inertia are seen in the action of dynamite when exploded upon the surface of a rock — the inertia of the air being sufficient to cause the rock to be pulverized by the sudden pressure; in the method of supplying locomotives with water while running at full speed, and in milling machinery in which rapidly revolving steel bars beat the grain to powder.

4. Fundamental Units and Measurements. The measurement of any concrete or physical quantity consists in comparing it with some quantity of the same kind assumed as the standard or unit. Its magnitude or measure is then stated in terms of that unit, and consists of two parts: a *numerical* part, and the part which *names the unit* with which it has been compared. Both these parts are needed to give an exact idea of the quantity in question. Thus we may give the length of a table as 4.57 *meters*, or as 4.57 *feet*, but no idea of the length of the table is possible until the *unit of length* is stated.

The fundamental concepts of Physics are those of space, mass and time, and most physical quantities may be expressed in terms of these. For this reason the units of *length*, *mass* and *time* are called *fundamental units*, and all other units expressible in terms of these are called *derived units*. Such a system of units is called an absolute system. In the system in common use among scientific men the unit of length is the *centimeter*, the unit of mass is the *gram* and the unit of time is the *second.* This is usually called the C. G. S. system. The corresponding units in the English system are the *foot*, the *pound* and the *second.*

The *centimeter* is the one hundredth part of the standard meter. The standard meter is represented by the distance, at the temperature of melting ice, between two marks on a certain bar of platinum-iridium, known as the international meter, kept at the International Bureau of Weights and Measures, near Paris. Two copies of this meter, known as the "national prototypes," are kept at the Bureau of Standards in Washington.

The meter was originally intended to be the one ten-millionth part of an earth-quadrant from equator to pole. Subsequent measurements have shown this distance to be 10,000,880 meters. The term *meter*, therefore, refers to the bar of metal and has no relation to the shape or size of the earth. By an act of Congress in 1866, the yard is defined as $\frac{3600}{3937}$ of a meter, hence the relation between the centimeter and the inch is

$$1 \text{ in} = 2.54 \text{ cm}$$

The *gram* is the one thousandth part of a mass of metal called a kilogram. The international kilogram is also kept by the International Bureau of Weights and Measures, near Paris. Two prototypes of this kilogram are kept by the Bureau of Standards in Washington.

It was intended that the gram should represent the mass of one cubic centimeter of distilled water at its temperature of maximum density [4° C]. Although more exact determinations have shown this relation to be slightly in error, yet for all practical purposes we may regard the mass of one cubic centimeter of distilled water at 4° C as equal to one gram. By the same Act of Congress in 1866, the relation between the kilogram and the pound was declared to be

$$1 \text{ kilo} = 2.2 \text{ lb}$$

The *second* is the unit of time employed in scientific measurements and may be defined as the $\frac{1}{86400}$ part of a mean solar day, where a mean solar day denotes the average time between the successive passages of the sun across the meridian, taken throughout the year.

It remains to be noted that while the units of mass, length and time are called fundamental units, there are employed other units not directly reducible to these, though connected with them by certain constants which have been determined by experiment. Such units are the units of temperature, of heat and of luminous intensity.

5. Dimensional Formulae and Derived Units. It is frequently of advantage to express physical quantities in general terms, in order to show more clearly their relation to each other. In such cases we write the symbols $[M]$, $[L]$ and $[T]$, with the proper exponents, to show how the fundamental quantities of mass, length and time enter into the derived units in question. Such a formula is called a *dimensional* formula. Since the dimensional formula is concerned with the *nature* of the quantity, rather than with its magnitude, numerical coefficients do not appear. A few examples of derived units will illustrate.

Area. The area of any plane figure is proportional to the product of two of its linear dimensions; hence the dimensional formula for *any area* is $[L^2]$; i.e. *the square of a length.* Unit area is 1 cm^2.

Volume. Since the volume of any solid is proportional to the product of three of its linear dimensions, the general formula for a volume becomes $[L^3]$; that is, *the cube of a length.* Unit volume is 1 cm^3 or 1 cc.

Density. The density of a body is, by definition, *the mass per unit volume.* It is found by dividing the mass of a body by its volume; hence, the formula for density is

$$\left[\frac{M}{L^3}\right] \text{ or } [ML^{-3}]$$

Unit density is a density of 1 g per cm^3.

Specific Volume. The specific volume of a substance is defined as *the volume containing unit mass of the substance.* It is, consequently, the reciprocal of the density of the substance, and its dimensional formula is, therefore,

$$\left[\frac{L^3}{M}\right] \text{ or } [L^3M^{-1}]$$

Unit specific volume is, accordingly, a volume of 1 cm^3 per gram.

6. Dimensional Formulae. Time Relations. Velocity. Velocity is *the time rate of motion.* If a body move over a space of s cm in t sec, then the time rate of motion, or average velocity, is given by the equation

$$v = \frac{s}{t} \qquad (1)$$

Even if the velocity vary from instant to instant, yet its value at any given instant is perfectly definite, and is obtained by dividing smaller and smaller spaces ds by the correspondingly small times dt, needed to traverse these spaces. The limiting value of this ratio, as s and t grow smaller and smaller,

$$v = \frac{ds}{dt} \qquad (2)$$

is, then, the time rate of motion for that instant of time.

Velocity includes the additional idea of motion in a definite direction. Thus, a velocity is stated as 10 cm per second, from north to south, or as 25 $\frac{\text{cm}}{\text{sec}}$ northeast. In cases where direction is either of no importance, or cannot be stated, this time rate of motion is termed *speed.* In this text, speed will be symbolized by $\bar{v}$. Since velocity is stated in units of length per unit of time, the dimensional formula becomes $[LT^{-1}]$. Unit velocity is a velocity of 1 cm per second.

Acceleration. Acceleration is the *time rate of change of velocity.* When the motion of a body is not uniform, the velocity is no longer constant, but changes every instant. This change in velocity may be a change either in magnitude or in direction. If this change in velocity be uniform, then the rate of change is constant, and is found by dividing the difference between the final and initial velocities, v and v_0, by the time t, during which the change in velocity occurs, or

$$a = \frac{v - v_0}{t} \tag{3}$$

Even if the acceleration be not constant, equation (3) gives the *average* acceleration *during the time t.* At any instant this time rate of velocity is given by the limiting value of this ratio, or

[illegible]

of change

of

$$a = \frac{dv}{dt} \tag{4}$$

The dimensional formula for acceleration is

$$\left[\frac{LT^{-1}}{T}\right] = [LT^{-2}]$$

Unit acceleration is an acceleration of 1 cm per second per second.

Momentum. Momentum is the quantity of motion possessed by a body, and is measured by the product of its mass and its velocity. Momentum is symbolized by mv, and its dimensional formula is $[MLT^{-1}]$. Unit momentum is possessed by unit mass moving with unit velocity.

Force. A body has no power to change its motion of itself. Any change in the motion of a body, either in magnitude or direction, must be due to some action upon the body, which we term a force. Force may be defined as that which changes the motion of a body, and is measured by the *time rate of change of momentum.* Its equation is

tend, to

$$F = \frac{mv - mv_0}{t} = ma \tag{5}$$

The dimensions of force are $[M L T^{-2}]$. The unit of force is the *dyne.* It is that force which will give *unit mass unit acceleration.*

7. Circular Measure of Angles. About the vertex C of the angle θ (Fig. 1), describe a circle with radius r, and denote the subtending arc, AB, by s. Then in circular measure the angle θ is defined by the equation

$$\theta = \frac{s}{r} \tag{6}$$

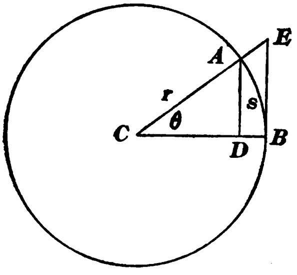

FIG. 1.

If θ be taken as unity, then $s = r$. This unit angle is called *a radian.* A radian is that angle whose subtending arc is equal to the radius. 1 radian = 57°.2958 = 3437′.75 = 206265″.

Again, from the point A drop a perpendicular to CB, and from B erect a perpendicular to CB to meet CA produced. Then, in the two right triangles ACD, and ECB, we have

$$\sin\theta = \frac{DA}{CA} \tag{7}$$

and

$$\tan\theta = \frac{BE}{CB} \tag{8}$$

Since CA and CB are radii of the same circle, we may write

$$\sin\theta = \frac{DA}{CB},\ \theta = \frac{BA}{CB},\ \tan\theta = \frac{BE}{CB},$$

or the angle θ lies between its sine and its tangent in value; that is,

$$\sin\theta < \theta < \tan\theta \tag{9}$$

As θ approaches 0, these values approach each other, and for small angles, we may write without serious error,

$$\sin\theta = \theta = \tan\theta \quad {}^{1)} \tag{10}$$

This approximation is frequently employed in physical formulae. The formulae are usually assumed as correct for values of θ under three degrees.

1) For experimental verification of this formula see Manual, Exercise 7.

7

~~8~~. Trigonometrical Formulae. The following trigonometrical relations will find frequent application. In any right triangle ABC (Fig. 2), right angled at C, we have by definition

$$\sin A = \frac{\textit{side opposite}}{\textit{hypotenuse}} = \frac{BC}{AB} \tag{11}$$

$$\cos A = \frac{\textit{side adjacent}}{\textit{hypotenuse}} = \frac{AC}{AB} \tag{12}$$

$$\tan A = \frac{\textit{side opposite}}{\textit{side adjacent}} = \frac{BC}{AC} = \frac{\sin A}{\cos A} \tag{13}$$

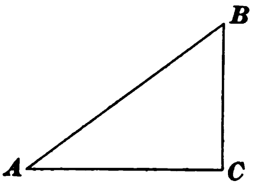

Fig. 2.

Also

$$\sin A = \cos (90° - A) = \cos B \tag{14}$$

$$\sin^2 A + \cos^2 A = 1 \tag{15}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \tag{16}$$

$$\sin 2A = 2 \sin A \cos A \tag{17}$$

The following table of values of the sine, cosine, and tangent of the various angles should be memorized.

TABLE I

θ	0°	30°	45°	60°	90°	180°
sine	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	0
cosine	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	-1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

9. Curvature. The direction of motion of a body at any point in a curved path is the tangent to the curve at that point. Since the direction in which the body is moving constantly changes, it is of importance to know the rate at which the direction of motion changes. *Curvature is defined as the space rate of change in direction.*

In the case of a point describing the circular arc PQ (Fig. 3), the change in direction is the angle included between the tangents at these points. This angle is in turn equal to the angle

θ, included between the radii OP and OQ. But by equation (6)

$$\theta = \frac{s}{r} \qquad (18)$$

hence the curvature is

$$\frac{\theta}{s} = \frac{1}{r} \qquad (19)$$

or the *curvature* at any point on a curve is the *reciprocal* of the *radius* of the *circle* which most nearly coincides with the curve at that point. In the circle the curvature is constant; it is variable in the ellipse or the parabola, and zero in a straight line.

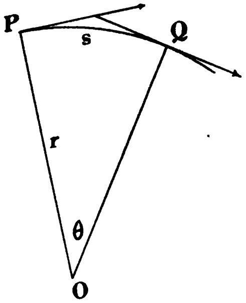

FIG. 3.

10. Vectors and Scalars. Certain physical quantities, such as displacements, velocities, accelerations, forces, etc., are of such a nature that they can be completely described only by giving their *magnitude, direction and sense.* Thus consider two elevator cars attached to the opposite ends of the same cable. While their speeds are exactly equal, yet their velocities are very different; although both cars move in the *same vertical direction* the velocity of the first may be 2 m per second *upward*, while the velocity of the second is 2 m per second *downward.* The *sense* of the velocities is given by the two terms, *upward* and *downward.* If the first velocity be taken as positive, the second must be considered as negative. Such quantities are called *directed quantities* or *vector quantities. A vector quantity is a quantity which may be completely represented by a straight line of definite length, traced in a specific sense.* Its magnitude is given by the length of the line, its direction by the direction of the line and its sense by an arrow-head showing the sense in which the line was traced.

Other physical magnitudes, such as mass, density, energy, power, electrical resistance, etc., have no relation to direction, and are completely described by stating their magnitude. Such quantities are called *scalar quantities.*

11. Projection upon Rectangular Axes. If we have a pair of rectangular axes XX', YY' (Fig. 4), then any vector may be resolved into two components parallel to the two given axes. Thus let OC represent such a vector making an angle θ with the axis of x. Then OA, the component of OC parallel to the axis of x, is defined by the equation

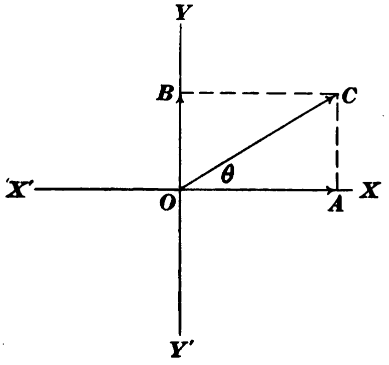

FIG. 4.

$$OA = OC \cos\theta \qquad (20)$$

Also

$$\begin{aligned} OB &= OC \cos(90^\circ - \theta) \\ &= OC \sin\theta \qquad (21) \end{aligned}$$

These two components, OA and OB, are called the *projections of* OC upon the axes of x and y respectively. The angle θ measures the difference in direction between the line OC and the x-axis and is called the direction angle. *To project any line upon any other, multiply the line in question by the cosine of the direction angle.*

12. Addition and Subtraction of Vectors. Given two vectors p and q, represented in magnitude, direction and sense by the straight lines AB and BC (Fig. 5), as indicated. Then their sum r, equal to AC, may be determined and calculated as follows: From the point C drop a perpendicular upon AB produced, and let the angle θ be the difference in direction between the two vectors. Then

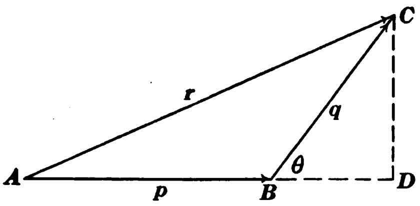

FIG. 5.

$$\overline{AC}^2 = \overline{AD}^2 + \overline{DC}^2 \qquad (22)$$

or
$$r^2 = [p + q \cos\theta]^2 + q^2 \sin^2\theta \tag{23}$$

whence
$$r^2 = p^2 + q^2 + 2pq \cos\theta \tag{24}$$

It is to be noted that this formula covers all possible cases arising from different values of θ. Thus, when $\theta = 0°$, $r = p + q$; for $\theta = 90°$, $r^2 = p^2 + q^2$; $\theta = 180°$, $r = p - q$.

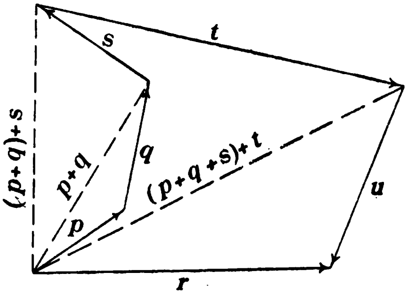

FIG. 6.

The solution may be effected graphically by laying off and connecting in order the lines AB and BC, representing the vectors p and q; then the line AC represents the vector sum of p and q, where the points A, B and C may be any points whatever.

13. Summation of any Number of Vectors. The foregoing graphical method may be extended to the case involving any number of vectors. Thus the resultant r (Fig. 6) of the vectors p, q, s, t and u is represented in magnitude, direction and sense by the line drawn from the beginning of the first to the end of the last, the vectors being added in any order whatever.

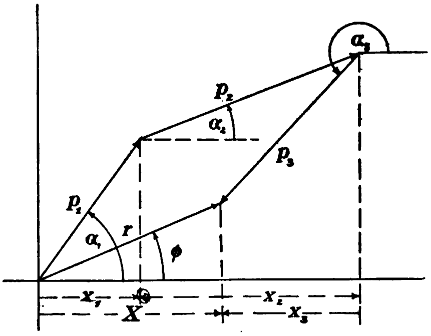

FIG. 7.

The numerical value of the resultant of any number of vectors is readily calculated by projecting the vectors upon the axes of x and y, adding the various components upon each axis, and then combining the x and y components. Thus, if (Fig. 7) $p_1, p_2, \cdots p_n$, be the various vectors, making angles α_1, $\alpha_2, \cdots \alpha_n$, with the axis of x, then the sum of the x component is

$$x_1 + x_2 + \cdots x_n = p_1 \cos\alpha_1 + p_2 \cos\alpha_2 + \cdots p_n \cos\alpha_n \tag{25}$$

or $$X = \Sigma p \cos \alpha \tag{26}$$

and similarly $$Y = \Sigma p \sin \alpha \tag{27}$$

whence $$r^2 = X^2 + Y^2 \tag{28}$$

and the angle ϕ between the resultant and the axis of x is given by the equation

$$\tan \phi = \frac{Y}{X} \tag{29}$$

Thus it is clear that the sum of any number of vectors is independent of the order in which they are added.

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MECHANICS OF SOLIDS

CHAPTER II

FORCE AND MOTION

14. Force. Our earliest ideas of force are derived from our sense of the muscular exertion needed to produce change in the motion of bodies about us. If we throw a stone into the air, roll a heavy truck along a smooth platform or move a log floating in the water, we are, in each case, conscious of a certain muscular effort needed to put these bodies in motion. Our experience also teaches us that this effort is greater in the case of large bodies than of small ones of the same kind, and also that it is more difficult to produce rapid motion in any case than to move the body slowly.

Again, we soon learn that this muscular effort is needed to *stop a body* when it is in motion. We learn to associate *rapid motion*, or the *motion of large bodies*, with *great muscular effort.* In short, we early learn that muscular effort is needed to *change the motion of a body.*

Whenever a body has its condition of motion changed, either in magnitude or in direction, this change is attributed to the action of something which we term *a force.* A force is thus considered as an action upon a mass, and is measured by the *product of the mass and the acceleration conferred upon the mass,* ***Force is defined as the product of mass and acceleration***, or

$$F = Ma \qquad (30)$$

The C. G. S. unit of force is the *dyne.* A dyne is that force which will give a gram mass an acceleration of 1 cm per second per second, or it is that force which will give to unit mass unit acceleration.

By an easy association of ideas, we come to attribute the motion of bodies to forces other than those directly due to our own muscular efforts. Thus, we say that the branches of the trees are tossed about by the force of the wind, that the motion of the falling body is due to the force of gravity, that the body attached to a spiral spring is kept from falling by the elastic force of the spring or that the ball is driven from the barrel of an air gun by the elastic force of the compressed air.

15. Pressure, Stress, Tension. When a force is distributed over an area, we are accustomed to specify the force exerted upon each unit of area. *Pressure denotes the force per unit area,* or

$$P = \frac{F}{A} \tag{31}$$

The dimensions of a pressure are, accordingly,

$$\left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}]$$

The pressure on the piston head of a steam engine is expressed in $\frac{\text{dynes}}{\text{cm}^2}$, or in $\frac{\text{lb}}{\text{in}^2}$.

At any point in the interior of a medium subjected to an external force, there exists a system of resisting forces which is termed a *stress.* A stress is likewise measured in terms of *force per unit area.* The dimensions of a stress are the same as those of a pressure.

It is frequently of advantage to discriminate between external and internal pressure. Thus a gas when compressed by an external force reacts against the compressing piston with a pressure equal to the external pressure. If the external forces acting upon a medium be directed toward each other, the medium is said to be under pressure, while if the forces be directed away from each other, it is said to be under tension.

Tension, like pressure, is usually measured in force per unit area, although the term *tension* is also used in a different sense in the case of *surface tension*, namely, as force per unit length. (See Art. 89.)

In all cases of stress, pressure or tension, the total force exerted upon any area is at once obtained by multiplying the value of the stress, pressure or tension by the area involved, as equation (31) clearly shows.

16. Impulse, Weight, and Inertia. When a force acts for but a short time, as in the case of a blow or a collision, the effect is called an *impulse*. An impulse is measured by the product of the force and the time during which the force acts, or

$$\text{Impulse} = Ft \tag{32}$$

The element of time is essential to the consideration of the effect of any force, since nothing short of an infinite force could produce an effect in zero time.

Again every force is to be considered as resulting from the mutual action of two bodies. Under this aspect there exists *a stress* in the medium *between the two bodies*. Thus a mass of 1 kilogram is attracted towards the earth and in turn attracts the earth with a force of 980,000 dynes. This mutual action tends to produce motion in the case of each body. The force by which a body is attracted toward the earth is called its *weight*, and its weight is the product of its mass and the acceleration due to gravity, or

$$W = Mg \tag{33}$$

Since the value of g increases slightly from the equator to the pole, it follows that the weight of a body is not constant at different points upon the earth. The *mass* of the body, however, *is constant*.

Weight is not to be confused with inertia, which signifies the resistance which a body offers to being set in motion *in any direction*. The weight of a body may be determined by means of the ordinary spring balance. When the body attached to the spring comes to rest, the force Mg, due to the mutual attraction between the body and the earth, is equilibrated by the elastic force due to a definite distortion of the spring. This force represents the *weight* of the body. If, now, the balance be suddenly given an acceleration *a*, upward, the *inertia* of the body, causes ~~appears in~~ an *increased stretch* in the spring, and this effect ~~is~~ measured by *Ma*. This force, due to inertia may be increased indefinitely as *a* is increased. Hence inertia has been defined as the *kinetic reaction* of matter against a change of motion. The inertia of a mass is a constant and characteristic property; its weight is a function of the acceleration due to gravity. man

17. Motion. Displacement is a change of position without regard to time. Motion is a change of position occurring in time. All motion is purely relative. Neither absolute rest nor absolute motion are known in the universe. Motion embodies the two concepts of space and time, and may be conveniently subdivided in accordance with these fundamental relations.

1. *Space Relations.* From the point of view of space relations motions may be said to be of two kinds: (*a*) motion of translation, (*b*) motion of rotation.

2. *Time Relations.* Under this aspect, motion may be studied in its relation to acceleration, from which we have: (*a*) uniform motion, (*b*) uniformly accelerated motion, (*c*) simple harmonic motion.

Besides these simple relations there exist very many combinations, only a few of which can be noticed here.

Space Relations. (*a*) *Translation.* If we imagine a particle to move in space, its path is a *line*, either straight or curved. Such motion is termed *linear motion*, and the displacement, velocity and acceleration concerned are *linear* in character in each case. Such motion is pure translation. If now an extended body move in such a way that each point in the body traces a *right line*, then the body is said to undergo *translation.* Examples of translation are seen in the up-and-down motion of an elevator, or in the motion of a train of cars on a straight level track.

(*b*) *Rotation.* If, on the other hand, the body move so that each point in the body describes a circle about a certain line, then the body is said to *rotate* and the motion is one of *rotation.* The line about which all the points in the body describe circles is called the ***axis of rotation.*** Since all these circles are described in the same time, it follows that the radii of these various circles all sweep out angle *at the same rate.* Rotation is therefore angular motion, and the displacement, velocity and acceleration concerned are all ***angular*** in character.

the Examples of rotation are seen in motion of the fly wheel of an engine, or in the spinning of the wheel of a bicycle when held free from the ground.

In nature these two kinds of motion are rarely found entirely distinct from each other. A stick or a ball when thrown into the air, undergoes both rotation and translation at the same time. Examples of this sort of motion are seen in the motion of a baseball when struck "foul," or in the motion of a carriage wheel as it rolls along the ground.

Time Relations. (*a*) *Uniform motion.* If a body undergo either translation or rotation under circumstances such that the acceleration is constantly zero, we have the condition for uniform motion. The motion may be either uniform linear motion or uniform angular motion.

(*b*) *Uniformly accelerated motion.* If translation or rotation occur under circumstances such that the acceleration has and maintains a constant value, we shall have uniformly accelerated motion, of translation or of rotation, as the case may be.

(*c*) *Simple harmonic motion.* In this type of motion the acceleration is directly proportional to the displacement from the position of rest of the body. The resulting motion is either linear or angular simple harmonic motion, according as the displacement from the position of rest was a linear or an angular displacement. To the study of this form of motion several subsequent articles will be devoted.

Besides the various forms of motion already mentioned, there exist in nature numerous combinations, some of which are extremely complex and much too difficult for treatment in an elementary text.

18. Newton's First Law of Motion. We have seen that a body is powerless to acquire motion of itself, and equally incapable of coming to rest of itself, if in motion. Change of motion, therefore, is always due to the action of a force. The equation

$$F = Ma$$

shows that force does not appear except in connection with matter. There is always a mass involved. Again the factor a indicates that force is exerted *only while the motion is changing*. A steam engine exerts force in pumping water from a well in that it sets the water in motion; the exploding gunpowder exerts force upon the cannon ball during the time the ball is passing from the breech to the muzzle of the gun. The ball in turn exerts force only while its motion is changing, *i.e.* while it is smashing the target or piercing the armor of the ship. A ball flying through space and encountering no resistance exerts no force; its motion is unchanged. A locomotive pulling a train with uniform velocity along a level track exerts force sufficient to overcome friction and no more. A body in motion moves until some force stops it. This is all summed up in Newton's first law of motion:

"*Every body continues in its state of rest or uniform motion in a straight line, except in so far as it is compelled to change that state by force impressed upon it.*"

This law is embodied in the equation

$$F = Ma$$

If a be zero, there is no force; hence there must exist either *rest* or *motion*. Again, a denotes the rate of change of velocity *either* in *magnitude or direction;* hence if a be zero, there must exist either *rest* or *uniform motion in a straight line.*

All apparent exceptions to the action of this law are in reality but proofs of this truth. A stone thrown into the air does not move with uniform velocity in a straight line for a single instant; yet the reason is found in the impressed force of gravitation, which compels it to change that state.

19. Newton's Second Law of Motion. The second law of motion is:

"*Change of motion is proportional to the moving force impressed, and takes place in the direction in which the force acts.*"

It is to be noted that by "change of motion" Newton meant *change in momentum*, as his own explanation of the meaning of the law clearly shows. He explains this law as follows: "If a force generate any motion, a double force will generate a double motion, a triple force a triple motion, whether they be applied simultaneously and at once, or gradually and successively. This motion, if the body were already moving, is either added to the previous motion, if in the same direction, or subtracted from it, if directly opposed, or compounded with it if the two motions are inclined at an angle."

This law is also embodied in the equation

$$F = Ma$$

and gives us a means of measuring either force or mass as the case may be. Thus we judge of the mass of a body by the force necessary to set it in motion. We kick a barrel lying on the ground to see whether it is empty or not. If full, it is started with difficulty; if empty, it moves very readily.

Again, suppose we have small cubical blocks of cork, aluminum, and lead, each mounted upon a little car so as to move readily upon a smooth table. We attach to each car a small spring balance and tie the balance to a rod by which we pull the blocks quickly along the table. The result will be that, while all the blocks have practically the same acceleration, the balances will indicate by their stretch the kinetic reactions, that is, the relative masses, of the various substances. In this case, the *acceleration* being the same for all, the *force* is *directly proportional* to the *mass*.

If, on the other hand, the blocks with their cars be placed upon a smooth table and be struck equal blows, as from a spring hammer, then the accelerations produced in the various blocks will afford a measure of their relative masses. The cork block would move off rapidly, the aluminum more slowly and the lead would scarcely be moved at all. We should conclude that the lead contains the most matter, the aluminum next, and the cork least. In this case the *force* is kept constant, and the *accelerations* vary *inversely* as the *masses*.

20. Newton's Third Law of Motion. The third law of motion states that

"*To every action there is an equal and contrary reaction, or the mutual actions of two bodies are equal and opposite.*"

In explanation of this law Newton adds, "Whatever presses or draws another body is pressed or drawn to the same extent by that body. If one press a stone with the finger, the finger is pressed by the stone. If a horse pull on a stone by a rope, the horse is pulled equally toward the stone; . . . and to the extent that the forward motion of the one is aided the forward motion of the other is impeded."

This law expresses the two-sided nature of every force. Force is always due to the mutual action of two bodies, the action of the one being equaled by the reaction of the other. This amounts to saying that forces always occur in pairs.

Again no force can be exerted unless there be some *resistance* to overcome. There can be no action unless there be something to act upon which will, in its turn, react. The athlete prefers to jump from a slab of stone; he cannot "rise" from a pile of straw or a heap of cushions. For the same reason it is tiresome to walk in melting snow or loose sand.

If motion ensue as the result of the action of two bodies, then the law expresses the equality of the resultant motions; that is, if a force confer upon the masses m and m', velocities of v and v', then the law states that their *momenta are equal*, or

$$mv = m'v' \tag{34}$$

Illustrations of this law are manifold. The explosive force of the powder drives the ball from the cannon; the two are shot apart, moving with velocities inversely as their masses. The recoil or "kick" of a gun is the greater the more nearly the masses of gun and projectile are made equal to each other.

The screw of a ship drives the water *backward* with a velocity as many times *greater* than the forward velocity of the vessel, as the mass of water moved is *less* than that of the vessel.

Boats have been propelled by machinery which pumped water in at the bow and expelled it in a small stream under high velocity at the stern. Again, the blades of the propeller of an aeroplane are much longer and rotate at a much higher speed than those of the propeller of a boat, since the volume of air to be displaced is much greater than the corresponding volume of water needed to furnish an equal reaction. The motions of fish in the water, and of birds in the air, the ascent of skyrockets and the action of rotary lawn sprinklers are all explained in accordance with Newton's third law of motion.

CHAPTER III

TYPES OF MOTION

21. Uniform Motion. The simplest type of motion is uniform motion. In the case of rectilinear motion the linear *velocity* v remains *constant;* that is, the space traversed in unit time and the direction and sense of motion all remain *unchanged.* Equal spaces are described in equal times. The *acceleration* is therefore zero. We may, from these conditions, write down the equation of uniform motion from definition. We have

$$\frac{s}{t} = v = constant \tag{35}$$

whence

$$s = vt \tag{36}$$

Similarly, if a body rotate uniformly about an axis, then a straight line drawn from the axis to any point in the body may be conceived as sweeping out angle about the axis at a uniform rate. If the *period*, or the time needed to describe a complete revolution, be T sec, then the total angle swept out in that time is 2π radians, and the time rate of generating angle is

$$\frac{2\pi}{T} = \omega \tag{37}$$

where ω is called the *angular velocity.* In uniform rotation the angular velocity is a constant. Hence if any angle θ be described in time t, then the angular velocity ω is defined by the equation

$$\frac{\theta}{t} = \omega = constant \tag{38}$$

whence the equation for uniform rotation becomes

$$\theta = \omega t \tag{39}$$

These conditions for uniform motion, whether linear or angular, involve the permanence of any motion once set up, and the absence of any force to maintain it.

22. Uniformly Accelerated Motion. In the case of uniformly accelerated linear motion the body moves with a constantly and uniformly increasing or decreasing velocity. It passes over unequal spaces in equal intervals of time. The velocity is no longer constant, but the *acceleration*, or the rate at which the velocity changes, *is constant.* Hence we may write

$$\frac{v - v_0}{t} = a = constant \tag{40}$$

The force producing uniformly accelerated motion is also a *constant force*, since

$$F = Ma$$

in which both M and a are constant.

From equation (40) we see that v, the velocity at any time t, is

$$v = v_0 + at \tag{41}$$

or the final velocity v is equal to the initial velocity v_0, plus the *change in velocity* acquired in time t.

The average velocity v' during this interval of time is, of course,

$$v' = \frac{v + v_0}{2} \tag{42}$$

where v' denotes the constant velocity at which the body would have described the same space in time t. The expression for the space described is readily found from equation (36); thus we have

$$s = v't = \frac{v + v_0}{2} \cdot t \tag{43}$$

or, replacing v by its value from equation (41), we have

$$s = v_0 t + \frac{at^2}{2} \tag{44}$$

Finally, combining equations (40) and (43) to eliminate t, we have

$$v^2 = v_0^2 + 2as \tag{45}$$

It is further to be noted that the acceleration may be either positive or negative; in the latter case the acceleration and the motion are oppositely directed, and the acceleration becomes a retardation. In their general form the equations of uniformly accelerated motion become

$$\begin{aligned} v &= v_0 \pm at \\ s &= v_0 t \pm \frac{at^2}{2} \\ v^2 &= v_0^2 \pm 2as \end{aligned} \tag{46}$$

23. Freely Falling Bodies. In the case of a freely falling body equations (46) become, on substituting for the general acceleration a, the acceleration due to gravity ($g = 980$ cm per second per second),

$$\begin{aligned} v &= v_0 \pm gt \\ s &= v_0 t \pm \frac{gt^2}{2} \\ v^2 &= v_0^2 \pm 2gs \end{aligned} \tag{47}$$

In case the body start from rest, v_0 is zero, and all terms containing it disappear, hence

$$\begin{aligned} v &= gt \\ s &= \frac{gt^2}{2} \\ v^2 &= 2gs \end{aligned} \tag{48}$$

It is to be noted that in case of a body starting from rest the velocity is proportional to the time, the space described is proportional to the square of the time, and the space described in the consecutive seconds varies as the odd numbers 1, 3, 5, 7, etc., as may be readily shown.

If a body be thrown vertically upward with an initial velocity v_0, the motion and the acceleration are oppositely directed and the lower sign is to be used in equations (47). The time of ascent and the height to which a body thrown vertically upward will rise are found by setting the final velocity, $v = 0$, the value it assumes at the highest point. Then we have

$$t = \frac{v_0}{g} \tag{49}$$

$$s = \frac{v_0^2}{2g} \tag{50}$$

These values of t and s are the same as would be required to produce a velocity v_0, in the case of a body falling freely from rest.

24. Diminished Acceleration. Atwood's Machine. If we wish to study the laws of a falling body experimentally, it is necessary to reduce materially the acceleration, as otherwise the motion is much too rapid to permit of accurate observation. This may be done in several ways. For example, the force of gravity acting upon a small mass may be applied to one or more large masses as well, in which case the resultant acceleration is correspondingly diminished. Thus suppose that a body of mass M be placed upon a perfectly smooth, horizontal table and have attached to it a light flexible cord passing over a smooth peg at the end of the table, and that from this cord there be suspended a small mass m. In this case the stretching force in the string will be that due to the weight of the mass m while the system is at rest, and this force will produce motion in the two masses M and m. If we denote the resulting acceleration by a, we may equate the two expressions for the force as follows:

$$mg = (M + m)a \tag{51}$$

whence

$$a = \frac{m}{M + m}\, g \tag{52}$$

In this way the value of a may be made what we will, and the motion rendered so slow as to allow us to study it at leisure.

In *Atwood's machine* the light flexible cord passes over a light wheel having a groove in its rim and mounted upon "friction wheels" so as to turn as freely as possible. Two equal masses M_1 and M_2 are hung to the ends of the cord and are in equilibrium in any position. If now there be placed upon one of the masses a small rider of mass m, then the resultant motion of the system is due to the force of gravity upon this small mass alone. If we set M equal to the combined masses M_1 and M_2, and neglect friction and the effect of the light wheel, we may compute the resulting acceleration a at once from equation (52), and verify the result by actually observing the spaces passed over in one, two, three, or four seconds respectively. In case the rider be removed at any time, the acceleration becomes zero from that instant, and the motion becomes uniform motion. By means of special devices this may be accomplished and the machine may be used to verify all the conclusions represented in equations (48).

25. Motion on an Inclined Plane. The inclined plane is another device for reducing the effect of gravity. Suppose a particle of mass m to slide without friction down a plane AB (Fig. 8), making an angle ϕ with the horizon; it is required to find the equations of its motion. It is to be observed that the acceleration due to gravity effective in producing motion down the plane is the component parallel to the surface of the plane. This component is readily found by projecting g upon the plane (Art. 11). The effective component is seen to be $g \cos (90° - \phi)$ or $g \sin \phi$, and the equations of (47) become

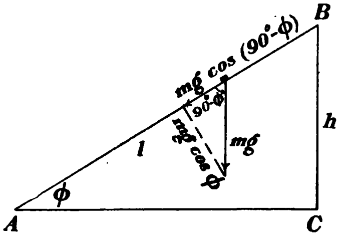

FIG. 8.

$$
\begin{aligned}
v &= v_0 \pm g \sin \phi t \\
s &= v_0 t \pm \frac{g \sin \phi t^2}{2} \qquad (53) \\
v^2 &= v_0^2 \pm 2\, g \sin \phi s
\end{aligned}
$$

Substituting for $\sin\phi$ its value h/l, and setting $v_0 = 0$, and $s = l$, in the last equation of (53) we see that

$$\bar{v}^2 = 2gh$$

or the velocity acquired by a body starting from rest and sliding down the plane is the same as that which it would have acquired falling through the vertical height h.

26. Uniform Circular Motion. A material particle describing circular motion at a constant speed is yet under the action of a constant force, since the *velocity* changes at every instant, not in *magnitude* but in *direction.*

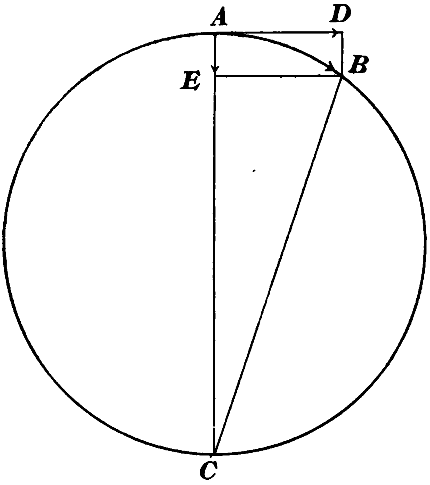

Fig. 9.

In uniform circular motion, therefore, there exists at every instant an acceleration toward the center. The value of this acceleration is readily calculated. Thus, in Fig. 9, a particle performing uniform circular motion describes the arc AB in time t, with a speed $\bar{v}$. It has in the same time been deflected from a straight line AD, through the distance AE, or AE is the space described due to the constant force acting toward the center. If $\bar{a}$ be the acceleration toward the center, then

$$AE = \frac{\bar{a}t^2}{2} \tag{54}$$

and

$$AB = \bar{v}t \tag{55}$$

since the motion is uniform.

If now the arc AB be taken very small, it will differ but little from a straight line, and the two triangles ABE and ABC are similar; hence

$$\frac{AB}{AE}=\frac{AC}{AB} \tag{56}$$

or

$$\overline{AB}^2=AE\cdot AC \tag{57}$$

whence

$$\bar{v}^2t^2=\frac{\bar{a}t^2}{2}\cdot 2r \tag{58}$$

or

$$\bar{a}=\frac{\bar{v}^2}{r} \tag{59}$$

If T be the time of a complete revolution of the particle in the circle, then

$$\bar{v}=\frac{2\pi r}{T} \tag{60}$$

and

$$\bar{a}=\frac{4\pi^2 r}{T^2} \tag{61}$$

whence

$$\bar{a}=\omega^2 r \tag{62}$$

27. Applications of Uniform Circular Motion. We have seen that in uniform circular motion a constant force is needed to keep the particle from flying off on a tangent. The force is termed *centripetal force*, and its measure is the product of the mass times the acceleration, or

$$F=m\frac{\bar{v}^2}{r} \tag{63}$$

The tendency of the particle to fly off, or the reaction of the particle against being pulled out of a straight line, is called *centrifugal force*, and its measure is also $m\bar{v}^2/r$. These two so-called forces are in reality but the two aspects of a *stress*, the one being the *action* of the moving mass, tending to move in a straight line, and the other the *reaction* of the center.

Examples of applications of these forces are seen in the action of a boy's sling; in the tendency of a bicycle rider to fall when turning a corner sharply while riding at full speed; and in the necessity for raising the outer rail in railway curves. The shape of the earth is an example of this action on the plastic mass of the earth while in rotation. Other applications of this principle are seen in machines for drying clothes; for separating honey from the honeycomb, or molasses from sugar. Cream separators, blood testers, lawn sprinklers, and the mechanical governors on steam engines all illustrate the same principle.

28. Simple Harmonic Motion. Fundamental Ideas. If from some solid support we suspend a spiral spring, in front of a mirror scale, and to the lower end of the spring attach a pan containing a small lead ball (Fig. 10), we shall find that the system will soon come to rest at some point whose position may be accurately read off on the scale by means of the pointer. If now there be placed in the pan masses of 1, 2, and 4 grams each, and the successive readings of the pointer recorded, it will be found that the displacement of the pan from its position of rest for 4 grams is twice the displacement for 2 grams and four times the displacement for 1 gram. In short the displacement is directly proportional to the force applied. Again, since the restoring force of the spring exactly balances the weights applied to it, it follows that for a given elongation, the restoring force is proportional to the displacement of the moving system, and since the mass of this system is constant, it also follows that the acceleration due to this force must be proportional to the displacement.

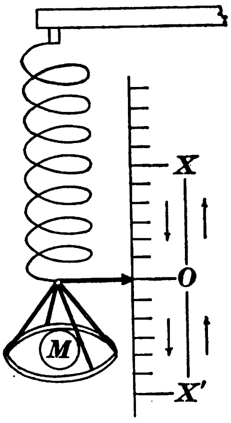

FIG. 10.

If, on a pair of rectangular axes XX' and YY' (Fig. 11), we let O represent the position of rest of the system, we may represent the downward displacements by spaces laid off on the negative ~~positive~~ axis of x, or OX, and upward displacements by corresponding spaces on OX'. Then if x represent the displacement of the system from O *at any time*, and a_x be the acceleration along the axis of x, we may express the relations described in the previous paragraph by the equation

$$a_x = -c \cdot x \qquad (64)$$

interchange

where the constant c represents the constant ratio between the acceleration a_x and the displacement x. The negative sign denotes that the *acceleration is always opposite in sense to the displacement.* Thus if the displacement of the spring be *downward* or *positive* then the acceleration due to the recoil of the spring is *negative* or *upward*, and *vice versa*.

If now we pull down the system till the displacement x reach any value, say 10 cm, and then release it, we shall have a regular, periodic, vibratory motion in a vertical line, above and below the point O (Fig. 10). The system in its vibratory motion traces the path $OXOX'O$ in the order indicated. Such a motion is a *simple harmonic motion. In simple harmonic motion the acceleration is proportional to the displacement from the position of rest and directed toward that position.*

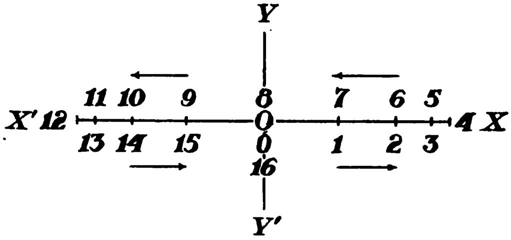

FIG. 11.

The maximum displacement of the system from the point O is called the *amplitude* a, of the vibration. The time elapsing between two successive passages through the same point in its path *in the same direction* is called the *period* T of the vibration.

If it were possible to take snap-shot photographs of the body, every $\frac{T}{16}$ sec during its entire vibration, the body would be seen at the points indicated by the figures on the axis, in the order shown by the arrows.

From the figure it is clear that the velocity of a point executing simple harmonic motion, is directed half the time in one direction and half the time in the other. Also that the velocity is a maximum at O, and zero at X and X', while the ~~*displacements in the succ*~~*essive equal intervals* of time, $\frac{T}{16}$ sec, counting ~~from O (Fig.~~ 11), are approximately proportional ~~to the num-~~

~~bers 4, 3, 2, 1, 1, 2, 3, 4, 4, 3, 2, 1, 1, 2, 3, 4,~~ in the ~~order and direction indicated in the figure~~.

In order to locate completely at any instant of time a body describing simple harmonic motion it is necessary to know the time which has elapsed since the body passed some fixed point, going in a definite direction. This elapsed time is called the *phase* of the motion, and is usually counted from the instant the body passes the position of rest *O*, going in the positive direction.

Examples of simple harmonic motion are seen in the vibrations of the prongs of a tuning fork, of guitar strings or of a pendulum bob, if the amplitude be small; other examples of approximately this form of motion are found in the motion of the piston rod of an engine, of the shuttle of a sewing machine or of the sickle of a reaping machine. Simple harmonic motion is the most important type of motion to be studied, as it finds its applications in sound, light and electricity as well as in mechanics.

29. Circle of Reference and Definitions. Simple harmonic motion may also be regarded as the apparent motion of a point describing uniform motion in a circle, when viewed at a great distance from the circle and in the plane of the circle. The apparent motion of the moons of Jupiter is a simple harmonic motion. These little bodies revolve about the planet in orbits nearly circular, and their motion as seen from the earth is an oscillatory motion about the planet as a center.

For purposes of study, simple harmonic motion is usually treated in connection with the related case of uniform motion in a circle. Thus if we consider a point moving uniformly round a circle (Fig. 12), in the direction of the arrow, then the projections of the point upon any diameter of this circle will represent a simple harmonic motion, upon that diameter. The projection will describe a complete to-and-fro vibration upon the diameter, while the point on the circumference describes a complete revolution.

Thus while the point on the circle moves from o' through C', B', A', to X, its projection falls successively upon O, c, b, a, and comes to rest for an instant at *X*. It then retraces its path to *O* as the point passes round to *o*. The point on the circle has described half a circumference and its projection has made half a vibration. As the point passes on through *D*, *E*, *F*, to *X'*, its projection swings through *d*, *e*, *f*, reaching the limit of its motion in *X'*, and returning completes the second half of its vibration as the point passes through *F'*, *E'*, *D'*, back to *o'*. The circle upon a diameter of which the simple harmonic vibration is supposed to occur is called *the circle of reference*.

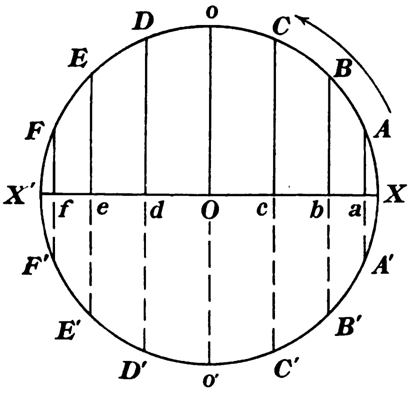

FIG. 12.

The *amplitude* of the vibration, that is, the maximum displacement from the center, now becomes the radius of the circle of reference. The *period* of the vibration, or the time required to make a complete to-and-fro vibration, now becomes the time required for the moving point to make a complete revolution in the circle.

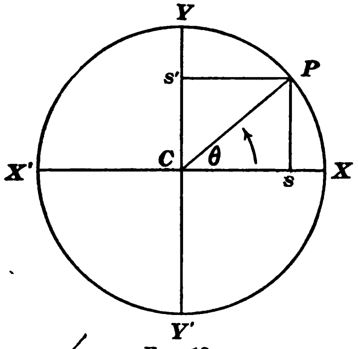

FIG. 13.

Phase is that fraction of a period which has elapsed since the moving point last passed through the position of rest in the positive direction. Phase may be expressed either in time, or in an angle which varies as the time.

30. Phase Relations. If ω represent the angular velocity of the radius vector *CP* (Fig. 13), then

$$\theta = \omega t$$

is the angle swept out in time *t*.

D

Set this figure on next page.

The angle θ is called the *time angle*, *i.e.* the angle swept out in time t, if time be counted from the instant the point P passes through X. In the case of motion on the axis of y, it is also the phase angle, since by definition, phase is measured from the instant at which the point P passes X, or its projection on the axis of y passes through C, in the positive direction.

For the same reason it is to be observed that in the case of motion on the axis of x, phase must be measured from the radius CY', since the point P is at Y' when its projection s, on the axis of x, passes through C going in the positive direction. In this case the phase angle is $Y'CP$, or $\theta + 90$. This shows that uniform circular motion is equivalent to two simple harmonic motions at right angles to each other, of the same period and amplitude, and differing in phase by 90°, or a quarter of a period.

(Fig. 13)

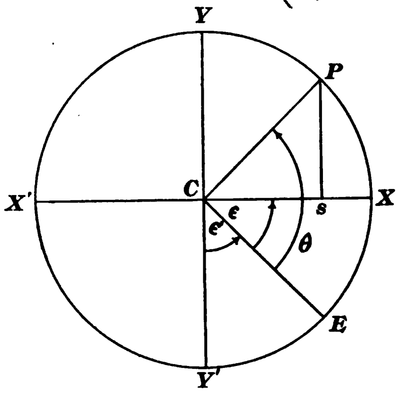

FIG. 14.

Again if we should begin to count time from some other point, as E (Fig. 14), then for the motion on the axis of y the time angle θ is ECP, while the phase angle is XCP as before, or $(\theta - \epsilon)$. For the motion on the axis of x the time angle is ECP, but the phase angle is $Y'CP$ as before, or $(\theta + \epsilon')$. The angles ϵ and ϵ' are called the *epoch angles*. The *epoch angle* is the angular difference between the time angle and the phase angle, or it is the angle which must be added to or subtracted from the time angle to produce the phase angle.

It is further to be noted that while ϵ and T are constant for any specific case, the time angle θ grows continuously from 0° to 2π while t grows from 0 to T.

31. Equations of Simple Harmonic Motion. In order to describe in mathematical terms the behavior of a point executing simple harmonic motion it is customary to express its distance from the position of rest in terms of an angle that varies as the time. Thus if CX or CY (Fig. 15) represent the amplitude r, then the motions of the points s and s' are completely described if we write

$$\left.\begin{aligned} x &= r \sin Y'CP \\ y &= r \sin XCP \end{aligned}\right\} \quad (65)$$

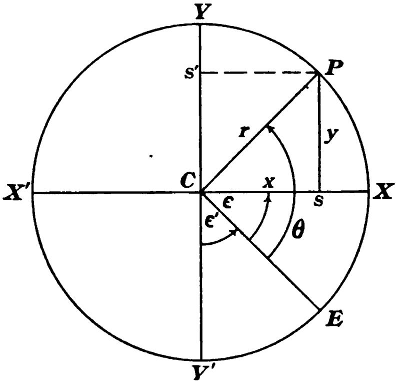

Fig. 15.

or

$$\left.\begin{aligned} x &= r \sin (\theta + \epsilon') \\ y &= r \sin (\theta - \epsilon) \end{aligned}\right\} \quad (66)$$

As a special case we may assume the point E to coincide with X, then $\theta = XCP$, $\epsilon = 0$, and $\epsilon' = 90°$, and equations (66) become

$$\left.\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}\right\} \quad (67)$$

Again, since the acceleration in uniform circular motion is

$$\bar{a} = \frac{4\,\pi^2 r}{T^2}$$

and *directed toward the center*, we have by projection upon the axes of x and y, the following expressions for the accelerations along these axes respectively:

$$\left.\begin{aligned} a_x &= -\frac{4\,\pi^2 r}{T^2} \cdot \cos \theta = -\,\omega^2 r \cdot \cos \theta \\ a_y &= -\frac{4\,\pi^2 r}{T^2} \cdot \sin \theta = -\,\omega^2 r \cdot \sin \theta \end{aligned}\right\} \quad (68)$$

The negative sign denotes that the acceleration is always opposite in sign to the displacement. Equations (68) show that the acceleration along the axis of x is a maximum when that along the axis of y is 0, and *vice versa*, since the one varies as the sine and the other as the cosine of the same angle.

If we substitute in equations (68) the values of $\sin\theta$ and $\cos\theta$, we have

$$\begin{aligned} a_x &= -\frac{4\pi^2 r}{T^2}\cdot\frac{x}{r} = -\frac{4\pi^2\cdot x}{T^2} = -\omega^2 x \\ a_y &= -\frac{4\pi^2 r}{T^2}\cdot\frac{y}{r} = -\frac{4\pi^2\cdot y}{T^2} = -\omega^2 y \end{aligned} \qquad (69)$$

which shows that the acceleration along the the axis of x is proportional to x, and that along the axis of y is proportional to y, or that *the acceleration is proportional to the displacement.* This is the characteristic of simple harmonic motion.

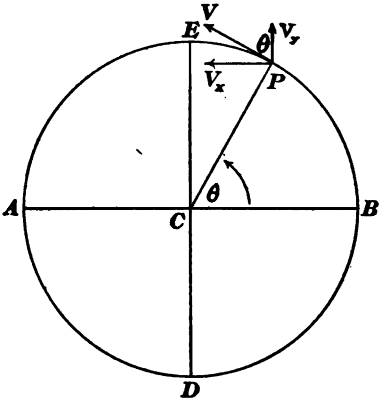

FIG. 16.

32. Velocity of a Point executing Simple Harmonic Motion. The velocity of a point executing simple harmonic motion may be readily calculated. Let P (Fig. 16) be the moving point, and let PV represent its velocity in the circle. Then V_x and V_y, the component velocities parallel to the axes of x and y, are found by projection as usual, and we have

$$\begin{aligned} V_x &= -V\cos(90^\circ-\theta) = -V\sin\theta \\ V_y &= V\cos\theta \end{aligned} \qquad (70)$$

But V is the velocity in the circle, and is therefore $\frac{2\pi r}{T}$, hence

$$V_x = -\frac{2\pi r}{T}\cdot\sin\theta \qquad V_y = \frac{2\pi r}{T}\cdot\cos\theta \qquad (71)$$

From equations (71) it appears that the velocity along the axis of x is a maximum when the velocity along the axis of y is zero, and *vice versa.* Also from comparison with (68) we see that when the velocity along the axis of x is a maximum, the acceleration along x is zero; this means the velocity is greatest at the middle of the swing where the acceleration is zero, and zero at the end of the swing where the acceleration is greatest.

It should also be noted that since the angular velocity ω is a constant, the time of a vibration, T, is also a constant, and the vibrations of a body executing simple harmonic motion *are all performed in the same time, and are independent of the amplitude.* Such vibrations are said to be ***isochronous***. This is seen in the constancy of the pitch of the musical note from a string or tuning fork, as the vibrations die away. This characteristic of simple harmonic motion is of high importance in the theory of the pendulum, in acoustics and optics.

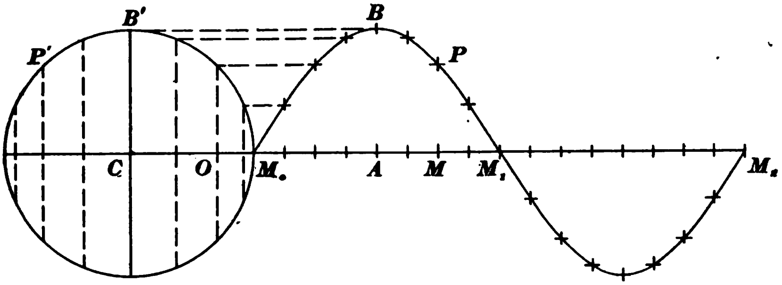

FIG. 17.

33. The Curve of Sines. An important aid to the study of simple harmonic motion is found in the graphical method, whereby the moving body is made to trace its path upon some recording surface. In such cases the simple harmonic motion is compounded with a uniform motion in a straight line and the resultant curve is called a ***sine curve***. Such a curve is readily constructed as follows: Divide each quadrant of the circle (Fig. 17) into four equal parts, each of which will thus correspond to the distance passed over by the point moving in the circle, in one sixteenth of a period. From the points of division drop perpendiculars to the x diameter; these perpendiculars will then denote the displacements of the point executing simple harmonic motion on the y axis, at the ends of the successive intervals of time, and are therefore proportional to the sines of the angles swept out in $T/16$, $2\,T/16$, $3\,T/16$, etc. Next lay off on the x diameter produced a series of *equal lengths* to represent the spaces described in the *same intervals*, due to the *uniform motion*, and beginning at M_0, erect perpendiculars equal to the y displacements at the corresponding times. Finally sketch a smooth curve through the extremities of these perpendiculars and the *sine curve* is the result.

Such a curve is readily obtained by allowing a tuning fork to trace, with a fine style, its vibrations upon a sheet of smoked glass or paper. The time interval between M_0 and M_1 corresponds to a half period, or the points at M_0 and M_1 *differ in phase* by half a period, while the points at M_0 and M_2 are in the the *same phase*.

The distance M_0M_1 is a *half wave length*, and M_0M_2 is a *complete wave length*, where a *wave length* denotes the distance the wave form has run forward in a single period T. Hence we have the important relation

$$\lambda = VT \tag{72}$$

where λ denotes the wave length, V the velocity of the wave motion, and T the period of the simple harmonic vibration.

Problems

1. A body has a velocity of 60 mi an hour. Find its velocity in feet per second, and in centimeters per second.

2. A body starts with a velocity of 640 cm per second, and in 10 min has a velocity of 3428 cm per second. Find the acceleration.

3. A mass of 5000 g moves with a velocity of 4 m per second. Find its momentum.

4. A mass of 600 g starts from rest and in 6 sec has a velocity of 36 m per second. Find the force.

5. A vessel contains 400 cc of sulphuric acid, density 1.8 g per cubic centimeter. Find the mass of the acid.

6. Detroit is 38 mi from Ann Arbor. How long will it take to travel this distance at the rate of 5 km per hour?

7. Express a mass of 65 lb in grams; a weight of 65 lb in dynes.

8. What force will in 10 min give a mass of 700,000 g a velocity of 124,000 cm per second?

9. What is the force of gravity on a body whose mass is 700 lb?

10. Find the mass of 465 cc of lead.

11. A wire guy rope, the stretching force on which is 12×10^8 dynes, makes an angle of 60° with the horizon. Find the vertical and the horizontal components of the stretching force.

12. Find the acceleration produced upon a mass of 4 g by a force of 36 dynes.

13. A force of 60 dynes acts upon a body for one minute and imparts to it a velocity of 900 cm per second. Determine the mass of the body.

14. An engine draws a cage of mass 3000 kilos up a shaft at a uniform speed of 10 m per second. Find the stretching force in the rope. What is the stretching force if the cage move with a uniform acceleration of 10 m per second per second?

15. An elevator starts to descend with an acceleration of 300 cm per second per second. Find the apparent weight on its floor due to a man whose mass is 75 kilos. What would be his weight with respect to the elevator, if it started to *ascend* with the same acceleration?

16. A steamer whose velocity in still water is 6 mi an hour starts directly across a stream whose velocity is 10 mi an hour. Find the velocity of the steamer in crossing; also when it makes an angle of 30° with the current down stream.

17. A gun of mass 3000 kilos, placed upon a smooth horizontal plane, discharges a ball of 30 kilos mass at an elevation of 30° to the horizon. Find the velocity of the gun's recoil in terms of velocity of the ball.

18. An inelastic mass of 900 kilos moving with a velocity of 30 m per second meets another equal and similar mass moving in the opposite direction, at 10 m per second. Find velocity of total mass after impact.

19. A certain force acts upon m units of mass, and at the end of a second the mass is moving at the rate of 32 ft per second. What velocity would be produced in 32 units of mass by the same force, in the same time?

20. How many dynes are required to give a mass of 50 kilos a velocity of 12 m per second, the force being supposed to act for exactly one second?

21. How many dynes are needed to give a gram a velocity of 9.81 m per second, if the force act for one second? What if it act for two seconds?

22. A body falls freely from rest for 15.6 sec. Find the final velocity and the distance traversed.

23. How long will it take a body to fall 650 ft, and what velocity will it acquire?

24. A body is thrown downward with a velocity of 874 cm per second. Required its velocity and position at the end of 20 sec.

25. A body is thrown vertically upward with a velocity of 827 cm per second. How long will it continue to rise, and how high will it rise?

26. A body is thrown vertically upward with a velocity of 697 cm per second. How long will it take to rise 195 cm, and what velocity will it then possess?

27. A ball is thrown vertically upward to a height of 150 ft. With what velocity did it leave the hand?

28. A mass of 876 g is attached to a spring balance which is carried upward at such a rate that the balance indicates 932 g. What is the acceleration of the motion?

29. A mass of 162 kilos hanging by a perfectly flexible cord drags a mass of 973 kilos along the top of a smooth table. What is the acceleration of the system, and what is the stretching force in the cord?

30. Two masses of 100 g each are hung by a flexible cord over a frictionless pulley. A mass of 10 g is placed upon one of the 100 g masses. Required the acceleration of the system and the stretching force in the cord.

31. Masses of 938 and 762 g respectively are hung by a flexible cord over a frictionless pulley. How far must the masses move in order to acquire a velocity of 325 cm per second?

32. A body slides down a smooth plane 326 cm long, inclined at an angle of 45° to the horizon. Find the time of descent and the velocity with which it reaches the bottom.

33. A mass of 200 g is constrained to move in a circle of 600 cm radius with a velocity of 240 cm per second. What is the centripetal force and the period of revolution?

34. The distance of the moon from the earth is 3.84×10^{10} cm, and the lunar month is approximately 27 da and 8 hr. What is the acceleration due to the earth's attraction at the moon?

35. If a skater describe a circle of 100 ft radius with a speed of 20 ft per second, find the inclination of his body from the vertical in order that he may maintain his equilibrium.

36. If the equatorial radius of the earth is 3963.3 mi, find the time of rotation necessary for a body at the equator to weigh nothing, assuming $g = 981$ cm per second per second for the earth at rest.

37. A mass of 1 g moves uniformly round a circle 40 cm in diameter at the rate of 24 revolutions a minute. Compute the force toward the center.

38. A mass of 1 g executes simple harmonic motion with an amplitude of 4 cm and a period of 0.5 sec. Find the force toward the center when the phase is $T/4$, $T/8$, and $T/2$, respectively.

39. Show that the spaces passed over, starting from the center, by a body executing simple harmonic motion, in the successive time intervals $T/16$, are approximately proportional to 4, 3, 2, and 1.

40. A horizontal shelf moves vertically with simple harmonic motion, with a period equal to one second. Find the maximum amplitude it can have so that objects resting upon it may remain in contact with it at its highest point. $g = 980$ cm per second per second.

CHAPTER IV

WORK AND ENERGY

34. Work. Work consists in changing a state of motion or a state of stress, in opposition to forces tending to resist such an effect.

Examples of work are seen (*a*) in the starting of a heavy car upon a smooth, level track; (*b*) in the compressing of a gas into a cylinder; (*c*) in the pumping of water into an elevated reservoir; (*d*) in the winding of a watch; (*e*) in the charging of a storage battery; (*f*) in the action of gravity upon a system composed of a wheel and axle, to which is attached a heavy weight by means of a cord wound round the axle.

When a body moves in the direction of the force acting upon it, the force is said to do work upon the body in giving it motion, as in the case of a freely falling body; if the motion be in opposition to the force, work is said to be done upon the body *against* the force, in giving the body a change in condition, or putting it in a state of stress; an example of this is seen in compressing gas into a cylinder.

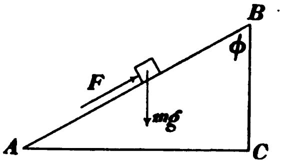

Fig. 18.

The measure of work done is the product of the *force* into the *distance*, in the direction of the force, or

$$W = Fs \tag{73}$$

If the motion take place in a line inclined to the direction of the force worked against, the effective displacement is found by projection, and the work is the product of the force into the effective displacement. An example of this is the work done against gravity in moving a body up an inclined plane. Thus the force F (Fig. 18), in working over the distance $s = AB$, is, in reality, lifting the mass through the distance CB. The effective displacement is therefore the vertical component of s, or $s \cos \phi$, and in this case the work is

$$W = mgs \cos \phi \tag{74}$$

therefore

$$F = mg \cos \phi \tag{75}$$

where ϕ is the angle between the direction of the motion and that of the force.

The C. G. S. unit of work is the *erg*. An erg is the work done by a force of one dyne acting through a distance of one centimeter; or unit work is done by unit force acting through unit distance. The force exerted by gravity upon a gram mass is 980 dynes. Therefore, to lift a gram mass one centimeter against gravity would require 980 ergs.

Since the erg is a very small unit, the joule $= 10^7$ ergs is generally used.

The dimensions of work are

$$[MLT^{-2} \times L] = [ML^2T^{-2}].$$

35. Work done by a Gas expanding under Constant Pressure. Let a mass of gas (Fig. 19) be inclosed in a cylinder, furnished with a frictionless piston of area a and mass m. Equilibrium will be established when the internal pressure of the gas just balances the external pressure, or when the upward force due to the stress in the gas equals the weight of the atmosphere upon the piston plus the weight of the piston. This force is

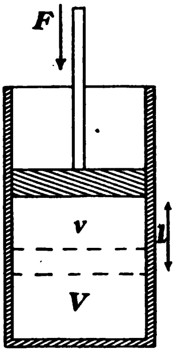

FIG. 19.

$$F = Ba + mg = Pa \tag{76}$$

where B is the atmospheric pressure, and P the pressure on the gas.

If now the gas be heated, it expands and pushes the piston forward through a distance l. The work done by the gas is therefore

$$Work = Fl = Pal = Pv \qquad (77)$$

where v is the change of volume produced.

The work done *by* a gas, expanding under *constant* pressure, is thus equal to the product of the pressure and the increase in volume. Conversely, if a constant pressure P produce a decrease v in the volume of a gas, the product Pv is the work done *upon* the gas.

In general the pressure of a gas is not constant while it is being compressed, but increases with the compression. In this case the above equation can be applied only for very small compressions during which the pressure may be assumed to remain constant.

36. Power. It is to be noted that the expression for work

$$W = Fs$$

contains no element of time. The same work is done in lifting a bag of grain to the top of a building, whether the work be done in an hour or in a month. *Power* involves the idea of the *rate at which work is done*, and may be defined as the *time rate of doing work.* Hence

$$Power = \mathscr{P} = \frac{W}{T} = \frac{Fs}{T} = \frac{Pv}{T}. \qquad (78)$$

The dimensions of power are $[ML^2T^{-3}]$.

The C.G.S. unit of power is the *watt.* A watt is the power which will do 10,000,000 ergs, or 1 *joule*, in one second. The practical unit of power is the *kilowatt*, or one thousand watts.

The English unit of work is the *foot pound*; that is, the work done in lifting a pound one foot high against gravity. The corresponding unit of power is the *horse power*, which denotes the power to do 33,000 foot pounds of work per minute, or 550 foot pounds per second.

One horse power is equal to 746 watts.

37. Energy. Whenever work has been done upon a system in producing a *change* either in its *motion*, its *position* or its *molecular condition*, the system has acquired the capability of doing work in turn.

Energy is the capability of doing work, possessed by a system by virtue of work having been previously done upon it. This is seen to be true of all the cases already cited (Art. 34). The car, once set in motion, can do work by virtue of that motion and to the extent of that motion; the wheel and axle and the falling weight each possesses energy, the one by virtue of its motion of translation, and the other by virtue of its motion of rotation; the gas compressed in the cylinder or the coiled spring of a watch each possesses energy due to a *stress* or a tendency to return to a former state. The water in the reservoir possesses energy due to gravitational stress; the chemical elements in a storage battery, having been separated by the electric current, now tend to reunite and thus possess energy due to chemical stress.

Energy is thus seen to exist in one of two distinct forms:

(*a*) **Energy of Motion,** *or Kinetic Energy;*
(*b*) **Energy of Stress,** *or Potential Energy.*

If we do work upon a system, we increase its energy, since we transfer energy from our bodies to the system. The sum total of energy in the system working and the system worked upon is at all times a constant quantity. When work is done by one system upon another, both kinds of energy are present, and there is a transfer of energy from the system working to the system worked upon. Hence to do work is to transfer energy from one system to another.

It is to be noted that whenever the motion is *with* the force, the motion of the system is increased and the system gains in kinetic energy, as in the case of a freely falling body. When the motion is *against* the force, kinetic energy is changed into potential, as in the case of a body thrown vertically upward.

Strictly speaking the system working is able to transfer only its available energy to the system worked upon. The working system may possess energy in the form of heat that is unavailable for the purpose of doing useful work. In the same way the system worked upon may receive energy from the system working in the form of heat, which escapes later, and is no longer available as energy in the system worked upon.

38. Expressions for Energy. Since by definition a system possesses energy only by virtue of work done upon it, the unit of energy is the same as the unit of work, *the erg.* The dimensions of energy are the same as those of work, $[ML^2T^{-2}]$.

Potential energy is stored up work, and its expression is

$$W = Fs = Mas = \textit{Potential Energy.} \tag{79}$$

To express the kinetic energy of a body in terms of its mass and velocity, we need to remember that we have a force F, acting through a space s, and the expression for kinetic energy may be deduced in a number of ways. Thus, if a mass m be acted upon by a force F, for a time t, during which it receives an acceleration a, it will pass over a space

$$s = \tfrac{1}{2}\, at^2,$$

and acquire a velocity

$$v = at.$$

Then the work done by the force will be

$$W = Fs = F \cdot \frac{at^2}{2} = \frac{ma^2t^2}{2} = \frac{mv^2}{2} \tag{80}$$

or

$$\textit{Kinetic Energy} = \frac{mv^2}{2} \tag{81}$$

Again, since

$$v^2 = 2\, as$$

we may write

$$mv^2 = 2\, mas \tag{82}$$

or

$$\frac{mv^2}{2} = mas = Fs = \textit{Kinetic Energy}$$

Finally we may deduce the same expression from fundamental definitions. Thus, the impulse is equal to the force into the time during which the force acts, or

$$Impulse = Ft = mv - mv_0 \tag{83}$$

also the mean velocity v', is

$$v' = \frac{s}{t} = \frac{v + v_0}{2} \tag{84}$$

whence, combining (83) and (84),

$$Fs = \frac{mv^2}{2} - \frac{mv_0^2}{2} \tag{85}$$

This last expression states that *the work* is equal to the *change in kinetic energy* produced, and is a more general expression for the work done upon a body. By setting v_0 equal to 0, we have

$$K.E. = \frac{mv^2}{2}$$

as before. Since kinetic energy like work is a scalar quantity and therefore independent of direction, we may substitute speed for velocity in the above formula.

39. Transformations of Energy. Transformations of energy occur on every hand. An excellent example is seen in the motion of a pendulum bob. At the highest point of the swing its energy is all potential, at the lowest point it is all kinetic; at intermediate points it is partly kinetic and partly potential. Were it not for the slight loss of energy in overcoming the resistance of the air and the stiffness of the cord, this transformation would go on forever.

Consider also the transformations of energy presented in the consumption of coal in the furnace of a steam boiler. The coal supply of the world represents the largest available source of potential energy and is simply the stored up sunshine of geologic ages. When the coal is burned in the furnace, this energy becomes kinetic in the form of heat, it appears as kinetic energy in the molecular motions of the water particles in steam, and as potential energy in the stress in the form of steam pressure on the boiler. In the steam engine the energy becomes the kinetic energy of the moving masses of the machinery and belts, which in turn may be transformed into light, motion, electric current or energy of chemical stress, and finally into heat again.

The transformation of energy from the potential to the kinetic form is *always a perfect one*, in that all the potential energy appears as kinetic. On the other hand the transformation from kinetic to potential is *never perfect;* some of the energy escapes as diffused heat and thus becomes unavailable for the purpose of doing useful work.

Potential energy tends to become a minimum. If in any system, any one of the stresses acting be removed, a redistribution of the energy occurs and the potential energy diminishes while the kinetic energy increases. An example of this is seen in the bursting of a reservoir full of water. The reaction of the restraining wall being removed, the water is carried down hill by the force of gravity with increasing velocity. The kinetic energy is increased at the expense of the potential. Other illustrations are seen in the bursting of a soap bubble, in the concentration of a dewdrop into a sphere, and in the position assumed by any body free to move into a new position of equilibrium. The result in all cases is that the system is at rest only when the potential energy is as small as possible.

40. Conservation of Energy. Throughout all the various transformations of energy it is to be noted that no body or system of bodies can acquire energy save at the expense of energy possessed by some other system. Hence we may say that to do work is to transfer energy from one system to another, and it seems certain, from the most careful experiments, that *the amount of energy lost by the one system is the exact equivalent of that gained by the other.* This means that no machine or combination of machines can ever be made to return more energy than is given to it. Perpetual motion is a delusion. Physically it is impossible to get something for nothing. Everything must be paid for in terms of energy.

Not only is it impossible to make a machine that will create energy, but no machine will ever return all the energy put into it. Owing to friction between the parts of the machine, some energy is transformed into heat and rendered unavailable for doing useful work. It is not lost, since energy, like matter, is indestructible.

The doctrine of the conservation of energy states that *in a system so situated that it neither loses energy from within nor gains energy from without, the amount of energy is constant.* No energy can be created, none can be destroyed.

Problems

1. How much work in ergs will be done by a force of 48 dynes acting through 24 cm?

2. What work will be required to lift 10 kg of water from a well 12.5 m deep?

3. The lower end of a ladder 16 m long stands on the ground at a distance of 273 cm from a wall against which the upper end rests. How much work will be done in carrying 30 kilos up the ladder?

4. The diameter of the cylinder of a steam engine is 18 in and its length is 24 in. What work in foot pounds will be done at each stroke of the piston if the average pressure of the steam be 110 lb per square inch?

5. If the above engine make 100 strokes per minute, calculate the horse power it will develop.

6. A shot of mass 2 kilos moving with a velocity of 200 m per second is just able to pierce a plank 4 cm thick. What velocity is required to pierce a plank 12 cm thick?

7. A stone of mass 5 kilos is thrown vertically upward with a velocity of 25 m per second. Find its kinetic energy at the end of two seconds.

8. A bullet of 100 g mass is discharged from a gun of mass 3 kilos, with a velocity of 400 per second. Compare the kinetic energies of bullet and of gun.

9. A ball of 25 kilos mass moves with a velocity 4 m per second. Compute its kinetic energy.

10. A man whose mass is 160 lb carries a hod and mortar of mass 75 lb from the ground to a scaffold 24 ft high, every 10 min. At what rate is this work done?

11. A standpipe 20 m high and 4 m in diameter is to be filled with water from a lake 8 m below the base of the standpipe. How long will it take a 10 H. P. engine to fill it?

12. A 100 gram mass is suspended from a spring balance which is carried in a balloon. What will be its apparent mass as shown by the index (*a*) when the balloon is ascending with a uniform acceleration of 240 cm per second per second? (*b*) when it is descending with an acceleration of 900 cm per second per second?

13. Two unequal masses are attached to the ends of a cord passing over a smooth peg. Find the ratio between them in order that they may move through 500 cm in two seconds, starting from rest.

14. The upper end of a smooth straight wire of length 100 ft, is attached to a pole 48 ft high. A bead is allowed to slip along the wire from top to bottom. Find its velocity on reaching the bottom. Also time of descent.

15. A ball thrown up is caught by the thrower 7 sec afterwards. How high did it go, and with what speed was it thrown? How far below its highest point was it 4 sec after the start?

16. The mass of a pendulum bob is 100 g, and the string is 1 m long. The bob is held so that the string is horizontal, and then allowed to fall. Find its kinetic energy when the string makes an angle of 30° with the vertical.

CHAPTER V

MECHANICS OF A RIGID BODY

41. Motion of a Rigid Body. A rigid body is one that suffers no change of form as a result of the forces acting upon it. When force is applied to a rigid body that is free to move, the body will acquire motion of translation or of rotation, or of both together. The motion of translation imparted to the body is fully accounted for by the equation

$$F = Ma$$

which shows that the acceleration imparted to the body will vary directly as the force, and inversely as the mass of the body.

If the angular velocity of a body change, this change must be attributed to the action of a force. Angular acceleration is the time rate of change of angular velocity. If this angular acceleration α be constant, then, by definition, we have

$$\frac{\omega - \omega_0}{t} = \alpha = constant \qquad (86)$$

If now a definite force be applied to a rotating body at any point, the resulting angular acceleration will vary greatly, according to the direction of the force and the distance of its point of application from the axis of rotation. Thus, in order to produce rotation, a force must have a component normal to the axis, and the farther its point of application is from this axis, the greater will be the angular acceleration produced. This change in angular velocity is due to the action of what is called the *torque* $\mathcal{T}$, or the moment of the force about the axis. Hence the torque, or moment of a force about any given axis, is that which changes, or tends to change, the state of rotation of the body with respect to this axis, and is measured by the product of the force times the lever arm, or it is the product of the force into the perpendicular distance from the axis of rotation to the line of action of the force. Thus (Fig. 20) the moment of the force about the axis through O is $PF \times OP$. Moments or torques are termed positive or negative, according as they tend to produce rotation in the counter-clockwise, or clockwise direction.

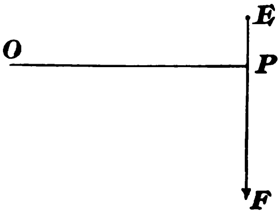

Fig. 20.

42. Resultant of Two Parallel Forces. Let F_1 and F_2 (Fig. 21) be two parallel forces applied to a rigid body in the directions $a'a$ and $b'b$; it is required to find the point of application C of the resultant R, such that its torque shall be equal to the sum of the torques of F_1 and F_2, or that it shall have the same effect in producing rotation about any point O as the combined effect of F_1 and F_2 taken together. From O drop a perpendicular upon the direction of F_1, F_2, and R, cutting them in A, B, and C. Represent OA, OB, and OC by x_1, x_2, and X respectively. Then, by the conditions of the problem,

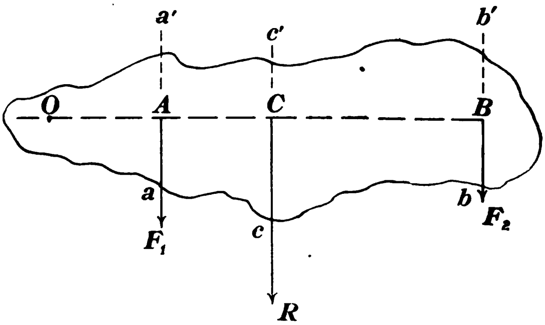

Fig. 21.

$$RX = F_1x_1 + F_2x_2 \tag{87}$$

But, by definition,
$$R = F_1 + F_2 \tag{88}$$

whence
$$X = \frac{F_1x_1 + F_2x_2}{F_1 + F_2} \tag{89}$$

or
$$\frac{F_1}{F_2} = \frac{x_2 - X}{X - x_1} \tag{90}$$

This shows that the point of application of the resultant divides the line joining the points of application of the two forces into two parts inversely as the forces. If the point O coincide with C, then

$$\frac{F_1}{F_2}=\frac{x_2}{-x_1} \tag{91}$$

or

$$F_2x_2=-F_1x_1 \tag{92}$$

which means that the moments of F_1 and F_2 about C are equal and opposite, and there is, therefore, no tendency to rotate about this point, or the torques about C are equal and opposite.

If F_1 be equal and opposite to F_2, then the value for X, from (89), becomes infinite, or the solution fails for the case of *forces equal* and *oppositely directed.* This simply means that *there is no point* at which a single force can be applied so as to produce equilibrium for this system. Such a system of forces is called a *couple. Two equal, parallel, oppositely directed forces constitute a couple*, and if applied to the adjacent parts of a rigid body, tend to produce rotation only. The moment of a couple is obviously the product of one of the forces into the perpendicular distance between them.

The only way by which a couple can be equilibrated is by means of another couple, of equal moment and oppositely directed.

43. Center of Inertia. If we take a body, as a long thin rod (Fig. 22), then every particle of matter in the rod is subject to the force of gravity, and the directions of these forces may be considered as sensibly parallel. If we take moments about any point O, situated anywhere, either *inside* or *outside* the rod, we find for X, the acting distance of the resultant R,

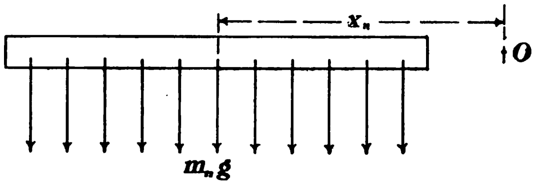

FIG. 22.

$$X = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + \cdots m_n g x_n}{m_1 g + m_2 g + m_3 g + \cdots m_n g} \tag{93}$$

or

$$X = \frac{\Sigma m g x}{\Sigma m g} = \frac{\Sigma m x}{\Sigma m} \tag{94}$$

The point defined by equation (94) is termed the *center of inertia* or the *center of mass*, of the rod. It denotes the point of application of the resultant of all parallel forces acting upon the rod. If a rigid support be placed under this point in the rod, all the forces of gravity acting upon the rod will be in equilibrium, since the sum of their moments about this point is equal to zero; therefore this point is frequently called the *center of gravity*.

Equation (94) also affords a means for determining the center of figure of any plane area A, made up of a large number of small areas, a_1, a_2, a_3, $\cdots$ a_n. If instead of mgx we substitute agh, and for mg we write ag, then H, the distance from any plane of reference, to the center of figure of the area, is given by the equation

$$H = \frac{\Sigma a g h}{\Sigma a g} = \frac{\Sigma a h}{\Sigma a} = \frac{\Sigma a h}{A} \tag{95}$$

This general principle may be applied to a few simple cases.

(*a*) The center of inertia of a line is at its middle point.

(*b*) The center of inertia of any lamina having an axis of symmetry, lies on this axis. If the lamina have two axes of symmetry, then the center of inertia of the lamina lies at their intersection.

(*c*) The center of inertia of a triangle lies at the intersection of the median lines of the triangle.

(*d*) The center of inertia of any polygon may be determined by dividing the polygon into triangles and considering the weight of the triangles as applied at their respective centers of inertia.

(*e*) The center of inertia of a triangular pyramid may be shown to be three fourths of the distance from the apex to the center of inertia of the triangular base of the pyramid.

44. Conditions of Equilibrium. Since the motion of a rigid body is either a motion of translation or of rotation, or a combination of the two, it follows that for the body to be in equilibrium, there should be no tendency to produce either translation or rotation. Hence the conditions for equilibrium are

$$\Sigma Fx = 0 \text{ (no rotation)} \qquad (96)$$

$$\Sigma F = 0 \text{ (no translation)} \qquad (97)$$

where moments and forces are to be taken with regard to their proper signs. The origin or center about which the moments are to be taken may be situated anywhere, as its position is of no importance.

45. Stability of Bodies. The principle of moments may be applied to the determination of the stability of bodies. A body is said to be *stable, or in stable equilibrium*, when it is necessary to do work upon it in order to displace it. In such a case the center of inertia must be raised, if the body is to be displaced, and the body, if disturbed, tends to return to its original position. If, on the other hand, the body on being disturbed has its center of inertia lowered, or tends to fall farther from its original position, it is said to be in *unstable equilibrium*. A book lying on its side on a smooth table is in stable equilibrium. If the book be set upon edge or upon end, it is in relatively less stable equilibrium. A ball or a cone lying on its side upon a smooth horizontal surface is said to be in *neutral or indifferent* equilibrium.

The stability of quadrupeds is great, on account of the broad base, represented by the area enclosed by the four feet, and because the center of inertia is almost vertically over the center of the base. For the kangaroo, though an apparent biped, the tail acts as a third leg, and the base is correspondingly increased. In man the base of support is small and the center of inertia is high, and the stability of the upright body is not great. In corpulent persons the increased effort to keep the center of gravity over the base leads to a swaying or rolling motion in walking, which is seen in an exaggerated form in the waddling walk of a duck or a goose.

46. Machines. A machine is any device for so transforming or transferring energy as to enable man to employ the forces of nature for doing useful work. In all cases a certain amount of energy is applied *to* the machine and in return a certain amount of useful work is done *by* the machine. An ideally perfect machine would return all the energy applied to it. In actual practice a portion of the applied energy is always expended in overcoming friction, and is thus frittered away as heat. Notwithstanding this loss, machines are employed for the purpose of applying to better advantage the forces at our command.

Since energy is work or force × space, it is evident that with a given amount of energy the force may be *increased* exactly in the ratio that the distance through which it acts is *decreased*. Thus if P be the force applied *to* the machine, and s the distance through which it acts, W the force exerted *by* the machine, and s' the distance through which the machine works, then, if we neglect friction, we have

$$Ps = Ws' \tag{98}$$

or

$$\frac{W}{P} = \frac{s}{s'} \tag{99}$$

The ratio W/P between the force exerted and the force applied is called the *mechanical advantage* of the machine.

Newton described six elementary machines to which all others may be reduced. These are the *lever*, the *pulley*, the *inclined plane*, the *wheel and axle*, the *wedge* and the *screw*. These may be reduced to three, since the wheel and axle is but a modified lever, and the screw and the wedge are but modifications of the inclined plane. In all cases, however, the law of machines holds good, viz.: *the force applied, multiplied into its acting distance, is equal to the force exerted multiplied into its acting distance.* A few examples will illustrate.

47. Simple Machines. *The lever.* The lever is a rigid bar turning freely about a fixed point F, called the fulcrum, which is to be considered as the center about which the moments of the forces P and W are to be taken.

For equilibrium these moments must be equal and opposite, or $\Sigma Fx = 0$

or $$P \cdot AF = W \cdot FB \qquad (100)$$

from which the *mechanical advantage* is

$$\frac{W}{P} = \frac{AF}{FB} \qquad (101)$$

in each case. According to the relative position of the fulcrum and the points of application of the forces P and W, levers are classified as follows (Fig. 23):

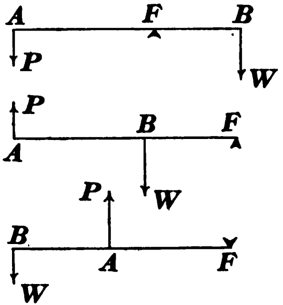

FIG. 23.

(*a*) *Levers of the first class;* fulcrum between P and W. Examples, a crowbar, a pair of scissors, or a pair of forceps.

(*b*) *Levers of a second class;* W between the fulcrum and P. Examples, a pair of nutcrackers, a wheelbarrow, or a door swung by its outer edge.

(*c*) *Levers of the third class;* P between W and the fulcrum. Examples, a pair of sugar tongs, or a pair of tweezers; a door swung by its inner edge; the human forearm.

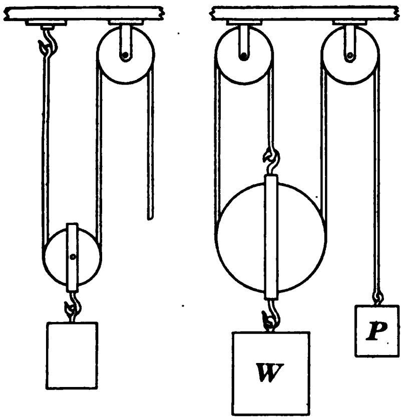

FIG. 24.

The pulley. A pulley is a small wheel turning freely about an axis, and having a grooved rim for the reception of a cord. A fixed pulley serves simply to change the direction of the applied force, ~~but offers no mechanical advantage~~. It is a lever with equal arms. A movable pulley (Fig. 24) permits of giving different velocities to P and W, and with a continuous cord its *mechanical advantage* is proportional to the number of strands of cord supporting the weight.

The inclined plane. In the inclined plane we have a plane surface inclined at an angle θ to the horizon. Three cases are possible:

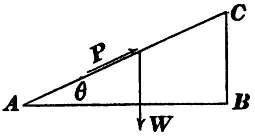

FIG. 25.

(*a*) *Force acts parallel to the plane.* In this case the force P (Fig. 25), tending to produce motion *up* the plane, is balanced by the component of the force of gravity *down* the plane, or for equilibrium

$$\Sigma F = 0$$

Hence we have

$$P = Mg(\cos 90^\circ - \theta) = Mg \sin \theta \tag{102}$$

If we represent the length of the plane by l, the height by h, and the base by b, we have, on substituting the value of $\sin \theta$ in the above expression,

$$P = Mg\frac{h}{l} \tag{103}$$

or

$$Pl = Mgh \tag{104}$$

This shows that the work done in pushing the mass M up the plane is the same as that required to lift it through the vertical height h, whence the *mechanical advantage* is

$$\frac{Mg}{P} = \frac{l}{h} \tag{105}$$

(*b*) *Force acts parallel to the base of the plane.* In this case the force P (Fig. 26) furnishes a component up the plane which must be balanced by the component of the weight down the plane, or

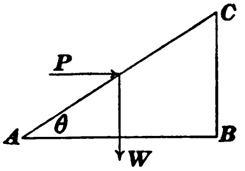

FIG. 26.

$$P \cos \theta = Mg \sin \theta \tag{106}$$

whence
$$P\frac{b}{l}=Mg\frac{h}{l} \tag{107}$$

or
$$P=Mg\frac{h}{b} \tag{108}$$

from which
$$Pb=Mgh \tag{109}$$

and again the work done in the two cases is equal. In this case we have for the *mechanical advantage*

$$\frac{Mg}{P}=\frac{b}{h} \tag{110}$$

(*c*) *Force acts at an angle ϕ with the plane.* In this case the equation of equilibrium is obtained as before by setting the components parallel to the plane equal to each other, or

$$P\cos\phi=Mg\sin\theta \tag{111}$$

The inclined plane permits of a small force being employed to do work by moving through a correspondingly greater distance. The winding curves of a railway as it makes its way up the mountain side illustrate the application of this principle.

48. Friction. Friction is the resistance offered to the motion of one portion of matter upon another. It is of the nature of a force. It seems to be due partly to the molecular attraction between the surfaces in contact, and partly to the nature and condition of those surfaces. Even the most highly polished surface is covered with minute scratches and projections which oppose the motion of a similar surface over it. Friction always acts tangentially to the surfaces in contact and always opposes the motion.

Friction is of two kinds: (*a*) *static friction*, or the resistance offered to a force tending to start one body to slide uniformly upon another; (*b*) *kinetic friction*, or the force necessary to keep the body sliding in uniform motion on a horizontal surface. Static friction is always greater than kinetic friction and varies from zero up to a maximum value, but is never more than that required to prevent motion.

The following facts concerning friction seem well established by experiment: (*a*) The friction depends upon the nature of the substances and the condition of the surfaces, but is independent of the area of the surfaces in contact.

(*b*) The friction is independent of the speed of the motion unless it be very small, when it rises to the maximum or static friction as the speed falls to zero.

(*c*) The friction is proportional to the force with which the surfaces are pressed together that is, it is that force times some factor which is always less than unity. This factor is called the *coëfficient of friction.*

49. The Balance. The balance is an instrument for the comparison of masses. It is a lever of the first class, having equal arms. In its simplest form it consists of a light, strong beam, supported at its center by a knife-edge resting upon an agate plate, and carrying upon knife-edges at its ends two pans for the reception of the masses to be compared. When disturbed the system oscillates about its position of equilibrium, to which it finally returns. When masses are placed in the pans, it is evident that the original position of equilibrium will be resumed only when the moments of the forces of gravity acting upon these masses are equal. If we assume the arms of the balance to be equal, we may set the masses equal to each other when their moments have been shown to be equal; that is, when the balance resumes its original position of equilibrium.

In case the arms of the balance are not of equal length, the true weight of the body may yet be determined by the method of double weighing. Let the arms of the balance be l and l_1, and let the true weight of the body be W. Suppose the body when hung from the arm l, to be balanced by a weight W_1 on l_1, and when hung from the arm l_1, to be balanced by a weight W_2 on l. Then by the principle of moments we have for the first case

$$Wl = W_1 l_1 \tag{112}$$

and for the second

$$W_2 l = W l_1 \tag{113}$$

Dividing the first equation by the second, we have

$$\frac{W}{W_2}=\frac{W_1}{W} \tag{114}$$

or

$$W^2=W_1W_2 \tag{115}$$

whence

$$W=\sqrt{W_1W_2} \tag{116}$$

From this it appears that the true weight of a body may be found upon a balance of unequal arms by weighing first in one pan and then in the other and taking the square root of the product. If W_1 and W_2 are not greatly different from each other, the arithmetical mean of the two weights is usually sufficient unless extreme accuracy is required.

50. Sensibility of the Balance. The delicacy of the balance is determined by its sensibility. This depends upon a number of considerations and is expressed in terms of the angular displacement of the beam for a small difference of weight p, usually one milligram, in the pans.

The sensibility of a balance is determined as follows: Let the points A, B, C (Fig. 27), represent the knife-edges of the two pans and the beam, and suppose these points to lie in a horizontal line. Let the center of gravity of the beam and pointer of the balance be at G, distant r from the center of suspension C, and let the weight of this system be W. For two equal weights P and P', placed in the pans, the system remains in equilibrium, provided the arms be of equal length. If an additional small weight p be placed in the right-hand pan, the system is displaced through the angle θ, and finally comes to rest in the position $A'CB'$. In this position the restoring moment due

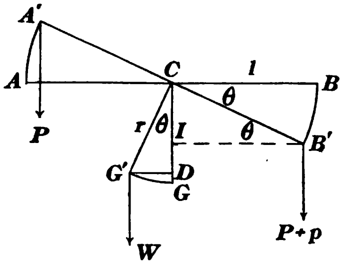

FIG. 27.

to W, and the disturbing moment due to p, must be equal and opposite. Then

$$W \times G'D = p \times B'I \tag{117}$$

Substituting for $B'I$ and $G'D$ their values in terms of θ, we have

$$Wr \sin\theta = pl \cos\theta \tag{118}$$

whence

$$\frac{\tan\theta}{p} = \frac{l}{Wr} \tag{119}$$

The quantity $\frac{\tan\theta}{p}$ is termed *the sensibility of the balance* 1) and is seen to vary directly as the length of the beam, and ? inversely as the weight of the ~~moving system~~, and as the distance of the center of gravity of the system, from the point of support.

In order, therefore, for the balance to be sensitive, the beam should be long and light and the distance r, between the center of gravity of the system and the center of support should be small. Practical considerations, however, set a limit to each of these theoretical suggestions. The beam must neither be so long nor so light as to render it liable to bend under the load it is to carry, else the sensibility would be diminished; also if the distance r be made too small, the system becomes unstable and the time of vibration becomes inconveniently long.

51. Moment of Inertia. It is now necessary to investigate the expression for the kinetic energy of a rotating body. Suppose a rigid body to rotate uniformly about a line through its center of gravity, O (Fig. 28), and let us call this line the axis of rotation. Also let ω denote the angular velocity. Then two particles of masses m_1 and m_2, situated at distances r_1 and r_2 from this axis, describe circle of circumferences $2\pi r_1$ and $2\pi r_2$

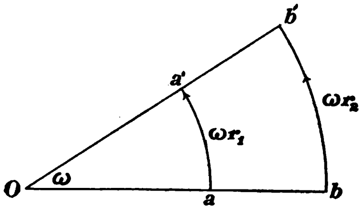

FIG. 28.

s

1) For specific directions in the use of the balance, see Manual, Exercises 12 and 13.

respectively, *in the same time*, T. The linear velocities v_1 and v_2 of the particles are consequently

$$\frac{2\pi r_1}{T} \text{ and } \frac{2\pi r_2}{T}$$

But since

$$\frac{2\pi}{T} = \omega$$

we have

$$\begin{aligned} v_1 &= \omega r_1 \\ v_2 &= \omega r_2 \end{aligned} \tag{120}$$

or, in general, the *linear velocity* of any point revolving about an axis is equal to the angular velocity times the radius; or,

$$v = \omega r \tag{121}$$

Again, the kinetic energies of the two particles m_1 and m_2 will be

$$\frac{m_1 v_1^2}{2} = \frac{m_1 \omega^2 r_1^2}{2}$$

and

$$\frac{m_2 v_2^2}{2} = \frac{m_2 \omega^2 r_2^2}{2} \tag{122}$$

If we suppose the rotating body to be made up of an indefinite number of particles, then the kinetic energy of the body will be the sum of the kinetic energies of all its particles, or

$$K.E. = \frac{m_1 \omega^2 r_1^2}{2} + \frac{m_2 \omega^2 r_2^2}{2} + \frac{m_3 \omega^2 r_3^2}{2} + \dots \frac{m_n \omega^2 r_n^2}{2} \tag{123}$$

or, since ω is constant for the entire body,

$$K.E. = \frac{\omega^2}{2} \Sigma m r^2. \tag{124}$$

The expression Σmr^2 is called the *moment of inertia*[1] of the body, and is designated by the letter I. It denotes the sum of the products, obtained by multiplying the mass of each individual particle by the square of its distance from the axis of rotation. This quantity, Σmr^2, has a perfectly definite, positive,

[1] For experimental determination of moments of inertia of a body see Manual, Exercises 31, 32.

numerical value for a definite body, rotating about a definite axis through its center of inertia. It will be observed that I depends, not only upon the *mass* of the body, but much more upon the *manner in which that mass is distributed with regard to the axis of rotation.* For every body it is possible to find a radius K, such that the mass of the body M multiplied by the square of this radius is equal to the moment of inertia, or

$$MK^2 = \Sigma mr^2 \tag{125}$$

Such a radius is called the *radius of gyration.*

The dimensions of a moment of inertia are $[ML^2]$ and the unit is gram centimeter square.

For convenience of reference the following table of moments of inertia is appended, where M denotes the mass of the body and I the required moment of inertia.

MOMENTS OF INERTIA (FORMULAE)

Uniform thin rod, axis through middle, length $= l$,

$$I = M\frac{l^2}{12} \tag{126}$$

Uniform thin rod, axis at one end, length $= l$,

$$I = M\frac{l^2}{3} \tag{127}$$

Rectangular lamina, axis through center and parallel to one side, length of sides a and b, side b bisected,

$$I = M\frac{b^2}{12}. \tag{128}$$

Rectangular lamina, axis through center and perpendicular to the plane, sides a and b,

$$I = M\frac{a^2 + b^2}{12} \tag{129}$$

Circular plate, axis through center perpendicular to the plate, radius $= r$,

$$I = M\frac{r^2}{2} \qquad (130)$$

Circular plate, axis any diameter, radius $= r$,

$$I = M\frac{r^2}{4} \qquad (131)$$

Circular ring, axis through center perpendicular to plane of ring, outer radius $= R$, inner radius $= r$,

$$I = M\frac{(R^2 + r^2)}{2} \qquad (132)$$

Circular ring, axis any diameter, radii as before,

$$I = M\frac{(R^2 + r^2)}{4} \qquad (133)$$

Moment of inertia about an axis parallel to an axis through center of gravity of body, and distant from same by a distance a, moment of inertia about axis through center of gravity $= I_0$,

$$I = I_0 + Ma^2 \qquad (134)$$

52. Moment of Inertia and Angular Acceleration. If α be the angular acceleration, then the linear acceleration

$$a = \frac{v - v_0}{t} = \frac{\omega r - \omega_0 r}{t} = \alpha r \qquad (135)$$

or the linear acceleration is equal to the angular acceleration times the radius; that is,

$$a = \alpha r \qquad (136)$$

If now we consider a particle of mass m, at a distance r from the axis of rotation, then the force necessary to give it a linear acceleration a is

$$F = ma = m\alpha r \qquad (137)$$

and the moment of the force is $m\alpha r \times r$, since the force acts tangentially to the circle described by m, or the moment *for a single particle* is $m\alpha r^2$.

The moment of the force necessary to give the entire body the same acceleration is $\Sigma m\alpha r^2$, or $\alpha\Sigma mr^2$; that is,

$$\textit{Moment} = \textit{Torque} = \mathcal{T} = I\alpha \qquad (138)$$

53. Kinetic Energy of Rotation. We have now derived two distinct formulae for kinetic energy.

$$(a) \qquad K.E. \text{ of translation} = \tfrac{1}{2} Mv^2 \qquad (139)$$

$$(b) \qquad K.E. \text{ of rotation} = \tfrac{1}{2} I\omega^2 \qquad (140)$$

On comparing these formulae for kinetic energy, we note from their symmetry that the angular velocity ω of a rotating body corresponds to the linear velocity v of a body undergoing translation; *also that the moment of inertia I of the rotating body corresponds to the mass M of the body undergoing translation.* This relation is most important, and finds constant application in mechanics. The flywheel of an engine acts as a reservoir of kinetic energy, not only on account of the mass, but still more on account of the distribution of its mass, since this is largely in the rim, thus making its moment of inertia as large as possible.

Again, it should be observed that a system may possess kinetic energy both of translation and of rotation at the same time. Thus the wheel of a bicycle when in motion possesses kinetic energy due to its motion of translation equal to $\frac{1}{2} M\bar{v}^2$, and also due to its rotation amounting to $\frac{1}{2} I\omega^2$. This fact has been ingeniously employed in the manufacture of steel shells for modern, long-distance, rifled cannon. The flying cylindrical shell possesses both energy of translation and energy of spin around its longer axis, due to the twisting motion imparted to it by the rifling of the gun. The striking end of the shell is now made of the hardest steel, and furnished with a screw point, so that the instant it strikes a target, the kinetic energy of rotation causes it to bore in like an auger, thus increasing its destructive action many fold.

The following table shows at a glance the striking analogy between the formulae relating to *linear* and *angular* motion:

TABLE II

Translation	Rotation	Translation	Rotation
s	θ	$s = v_0 t + \frac{1}{2} at^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
v	ω	$F = Ma$	$\mathcal{T} = I\alpha$
a	α	$W = Fs$	$W = \mathcal{T}\theta$
F	$\mathcal{T}$	$\mathcal{P} = Fv$	$\mathcal{P} = \mathcal{T}\omega$
M	I	$K.E. = \frac{1}{2} mv^2$	$K.E. = \frac{1}{2} I\omega^2$

54. Ideal Simple Pendulum. The ideal simple pendulum may be defined as a heavy material particle suspended by a weightless thread. As an approximation to this ideal we may use a lead ball supported by a fine silk cord. It is required to find an expression for the time of vibration of this pendulum. Let the pendulum of mass m be supported by a cord AB, of length l (Fig. 29). When drawn aside to the position B, it forms an angle θ, or BAN, with the vertical. The component of the weight mg, which is effective in producing motion, is $mg \cos (90° - \theta)$ or $mg \sin \theta$. Let NB equal x, the displacement, and let a_x be the acceleration toward the position of rest; then ma_x is the force urging the body toward N, or

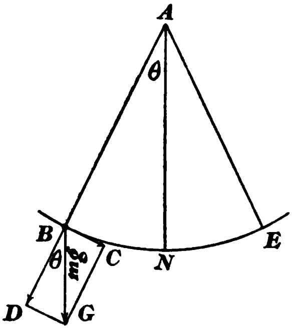

FIG. 29.

$$ma_x = - mg \sin \theta \tag{141}$$

whence

$$a_x = - g \sin \theta \tag{142}$$

also

$$\theta = \frac{x}{l} \tag{143}$$

If θ be small, we may set

$$\sin \theta = \theta = \tan \theta$$

therefore

$$a_x = - \frac{g}{l} \cdot x \tag{144}$$

Now since g and l are constants, it is clear that within the limits where we may write

$$\sin\theta = \theta = \tan\theta$$

the *acceleration is proportional to the displacement*, and the motion of the pendulum will be ***simple harmonic motion.*** But from equations (69) we have

$$a_x = -\frac{4\pi^2}{T^2}\cdot x$$

therefore
$$\frac{4\pi^2\cdot x}{T^2} = \frac{g}{l}\cdot x \tag{145}$$

or
$$T = 2\pi\sqrt{\frac{l}{g}} \tag{146}$$

This is the *period*, or the time of a *complete vibration*;[1] for a half vibration

$$T_1 = \pi\sqrt{\frac{l}{g}} \tag{147}$$

To find the length of the seconds pendulum, where

$$g = 980\,\frac{\text{cm}}{\text{sec}^2},\ \text{set}\ T_1 = 1,\ \text{or}\ 1 = \pi\sqrt{\frac{l}{980}}$$

from which
$$l = \frac{980}{\pi^2} = 99.3\ \text{cm}$$

55. Compound or Physical Pendulum. Any body suspended so as to swing freely about an axis forms a compound or physical pendulum. In such a pendulum the mass of the body may be considered as concentrated at its center of gravity. Suppose the heavy body (Fig. 30) be drawn aside through an angle θ, then the moment of the restoring force is $Mgh\sin\theta$, where h is the distance from the point of suspension to the center of gravity of the pendulum. This same moment tends to produce an angular acceleration a, and we have seen (Art.

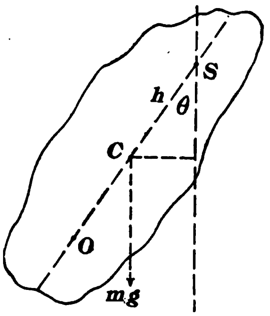

FIG. 30.

[1] For experimental verification of this formula see Manual, Exercise 24.

52) that such a moment is equal to $a\Sigma mr^2$, hence equating these two values for the moment of the force Mg upon the pendulum, we have

$$a\Sigma mr^2 = Mgh \sin \theta \tag{148}$$

whence

$$\frac{\Sigma mr^2}{Mh} = \frac{g \sin \theta}{a} \tag{149}$$

Now $\frac{g \sin \theta}{a}$ is a linear acceleration divided by an angular acceleration, and we have seen (Art. 52) that the quotient is *a length;* hence $\frac{\Sigma mr^2}{Mh}$ is also a length; as a matter of fact, it is the length of an equivalent simple pendulum, which will vibrate in the same time as the body. If we let

$$l = \frac{\Sigma mr^2}{Mh} = \frac{I}{Mh} \tag{150}$$

equal the length of this simple pendulum, we may write the expression for the time of vibration of the body at once.

$$T = 2\,\pi\sqrt{\frac{l}{g}} = 2\,\pi\sqrt{\frac{I}{Mhg}} \tag{151}$$

This is a general expression for the time of vibration of any heavy body when disturbed, about its position of equilibrium.

The length of a compound pendulum may be determined experimentally by suspending near it a simple pendulum and altering the length of the latter until the two vibrate in the same time. The length of the simple pendulum so found is

$$l = \frac{\Sigma mr^2}{Mh} = \frac{I}{Mh}$$

If now we lay off this length upon the compound pendulum, measuring *from* the center of suspension *through* the center of gravity, we find the *center of oscillation O*; that is, the center of oscillation is distant from the center of suspension by the length l, as defined by equation (150), and the pendulum behaves as if all the mass were located at this point.

It may be also shown that the center of oscillation and the center of suspension are interchangeable; or, in other words, for every point of suspension S there exists a conjugate point O, such that the pendulum will vibrate in the same time, whether suspended from S or from O.

The center of oscillation is also termed the *center of percussion*, since, if the pendulum be struck a blow at this point, it will move forward in its arc, with no jerk or jar upon the point S, as if the whole mass had received the blow. At any other point the effect of a blow would be to start the pendulum to swinging, and also to jerk it from its support owing to the tendency to rotate about some axis other than the point of support. This is well illustrated in the case of a baseball bat, which is to be considered as a compound pendulum. If the ball be received upon the bat at the center of percussion, no jarring of the hands is experienced by the striker. If the ball strike the bat at any other point, either too near the end or too near the hands, the jarring effect is very painful, and the bat is usually broken as well.

Problems

1. A weight of 350 kilos is carried by a bar, AB, 4.7 m long, supported at the ends. If the distance of the weight from the end A be 2 m, what is the weight borne by each support?

2. A uniform lever 8.2 ft long weighs 15 lb. If 26 lb be hung from one end and the bar be sustained by a support 2.6 ft from this end, what weight must be hung from the other end for the system to be in equilibrium?

3. It is found that a lever cut from a uniform bar weighing 4.2 lb to the foot balances at a distance of 2.3 ft from one end when weighted at this end with 5.4 lb. What is the length of the bar?

4. When a lever, AB, is supported at its center of gravity, it is found that a weight, W, hung at A will balance 2.5 lb at B; but when W is hung at B, it requires a weight of 19 lb at A to keep it in equilibrium. What is the weight W?

5. Find the magnitude and point of application of the resultant of two parallel forces, in the same direction, equal, respectively, to 25 and 42, when their lines of action are 3.4 ft apart.

6. A meter scale (of which the mass may be neglected) has suspended from the 10 cm mark, and from the 90 cm mark masses of 2 and 9 kilos respectively. Where must the scale be supported for equilibrium?

7. A lever is 20 cm long and its mass 15 g. Where must the fulcrum be placed in order that a mass of 10 g at one end may just balance a mass of 16 g at the other end?

8. A uniform, thin rod one meter long, whose mass may be neglected carries four masses of 4, 7, 10 and 22 g. The 4 g mass is at one end and the 22 g at the other, while the 7 and 10 g masses are 55 and 75 cm from the 4 g mass. Find the center of inertia of the system.

9. From a square plate of uniform thickness, one triangle formed by the intersection of the diagonals has been removed. Find the center of inertia of the remainder.

10. Masses of 7, 9 and 11 kilos are placed at the points A, B and C of a right triangle, right angled at C, of which the base AC, is 10 cm and the altitude BC, is 15 cm, and whose mass may be neglected. Find the center of inertia of the system.

11. A wheel of mass 100 lb is attached to end of a straight uniform axle 4 ft long and of mass 50 lb. Find the center of inertia of the system.

12. In a system of one fixed and one movable pulley (Fig. 24), what weight can be lifted by a force of 20 lb weight, neglecting friction and mass of pulley?

13. In a system of two fixed and one movable pulley (Fig. 24), what mass at end of cord will equilibrate 270 kilos attached to movable pulley, neglecting friction and mass of pulley as before?

14. What force acting horizontally can keep a mass of 16 kilos at rest on a smooth inclined plane, whose height is 3 m and base 4 m?

15. Find the force normal to the plane mentioned in problem 14.

16. A body of mass 25 kilos rests upon an inclined plane, which makes an angle of 30° with the horizon. Find the force necessary to produce equilibrium, (*a*) when the force acts parallel to plane, (*b*) when parallel to base.

17. A boy who can exert a force equal to 50 lb weight, wishes to roll a barrel of mass 200 lb into a window 4 ft high. How long a plank is required?

18. If a mass weigh 29.62 g, in one pan of a balance and 28.71 g in the other, what is its true mass?

19. If a mass of 625 g moving with a velocity of 786 cm per second meet a mass of 164 g moving in the opposite direction with same speed, what will be their velocity after impact, supposing the bodies to be inelastic?

20. A grindstone of radius 20 cm and density 2.2 g per cubic centimeter is 10 cm thick. Calculate its moment of inertia when rotated about an axis through the center.

21. If this stone rotate 5 times in 4 min, compute its kinetic energy.

22. Compare the moment of inertia of a meter rod of mass 150 g, when rotated about an axis, (*a*) through one end, and (*b*) through the center, both axes being normal to the length of the rod.

23. A board 2 cm thick and 8 cm long by 5 cm wide, of density 0.75 g per cubic centimeter, rotates about an axis, through its center of inertia and normal to its plane. Compute its moment of inertia.

24. A grindstone of mass 25 kilos and radius 40 cm rolls without sliding down an inclined plane 200 cm long and 50 cm high. Compare its kinetic energies of rotation and translation.

25. Find the time of vibration of a simple pendulum 20 cm long, where g equals 980 cm per second per second.

26. What would be the value of g, if the period of a simple pendulum 97.31 cm in length were 1.975 sec?

27. A pendulum loses 20 sec per day, where g equals 980.3 cm per second per second. Find its length.

28. A compound pendulum is composed of a wire 60 cm long, whose weight may be neglected, suspended from one end, and carrying masses of 3, 5 and 7 kilos at distances of 20, 40 and 60 cm from the point of support. Compute its length and time of double vibration.

29. A thin hoop of radius 30 cm and mass 100 g, hangs over a fine nail and vibrates in its own plane. Find its time of vibration.

30. A hollow cylinder rolls down a perfectly rough inclined plane in 10 min. Find the time a uniform solid cylinder would take to roll down the same plane.

31. The masses in an Atwood's machine are 355 and 345 g. In the *second* second, starting from rest, the heavier mass descends through 27 cm. What value does this give for g?

CHAPTER VI

ELASTICITY

56. Stress and Strain. Elasticity is that property by virtue of which matter resists the action of a force tending to change its shape or bulk, and which causes it to resume its original shape or bulk after the force is removed. If a body possess elasticity of shape, it is called *a solid;* if it possess no elasticity of shape, it is called *a fluid.* If a body have its size or shape changed under the action of a force, it is said to have been *strained.* The change in size or shape is called *a strain*, and is measured by the *relative change* so produced. Thus, if a mass of gas of volume V have its volume changed by a small amount dV, then the strain is $\frac{dV}{V}$; or if a solid of length L have its length changed by dL, then the strain is $\frac{dL}{L}$.

The external force applied to produce a strain is called the *load*, and, as a result of the strain produced, there arises in the strained medium a system of *resisting forces* which tend to produce equilibrium. This reaction of the medium against further deformation is called *a stress.* The stress at any point in a medium is measured by the *force per unit area.* After equilibrium ensues the load and the resisting force are equal. Hence stress may be evaluated in terms of the *applied force per unit area.* In the case of a compressed gas this stress is expressed as the *pressure* exerted by the gas in $\frac{\text{dynes}}{\text{cm}^2}$, or $\frac{\text{lb}}{\text{in}^2}$.

Fluids possess *perfect elasticity of bulk;* that is, they return exactly to their original bulk on removal of the compressing load. For every solid there is a limiting distortion beyond which the body when freed from the distorting load no longer completely regains its former shape. This distortion is called *the limit of elasticity.* In engineering practice the *elastic limit* is expressed in terms of the *stress* producing this limiting distortion.

All solids do not recover their initial shape with equal promptness. In some cases this return is much retarded, especially after repeated or long-continued distortion. This is commonly termed *elastic after effect*, and is quite noticeable in metals.

57. Hooke's Law. When an elastic body is distorted within its limit of elasticity, the stress called out by the distortion, tending to restore the body to its original condition, is *proportional to the distortion.* This is known as *Hooke's law*, and as originally stated, "*ut tensio sic vis*," expresses the proportionality between the stress and the strain. The applications of this law are very numerous, including every form of elastic reaction against strains produced by external mechanical agencies.

As will be readily seen, the application of Hooke's law involves the fundamental assumption that the force, and therefore the *acceleration*, is proportional *to the distortion.* If the original position of the system be taken as the position of rest, we have at once the characteristic condition of simple harmonic motion. Such motions are seen in the vibrations of elastic springs, of the air particles in the production of sound, in the varying charges in an electric condenser, and in compressional and torsional waves in solids.

From Hooke's law we see that in all cases where a body is distorted within its limits of elasticity, we have a relation of the form

$$\textit{stress} = c \times \textit{strain} \tag{152}$$

where *c* is a *constant* which is independent of either the stress or the strain, but which is characteristic of the substance undergoing distortion. It is called *a coefficient of elasticity.*

58. Coefficients of Elasticity. In general, twenty-one coefficients would be necessary to express completely the elastic nature of a body. If, however, the body be isotropic, these twenty-one coefficients reduce to two: the *coefficient of volume elasticity e, and the coefficient of rigidity, n.* The general expression for these coefficients is therefore the quotient arising from *dividing the stress by the strain.*

In the coefficient of volume elasticity e we have the stress, or applied pressure p, divided by the compression produced, where *compression* denotes the *change in volume v, divided by the original volume V*, or

$$e = \frac{p}{\frac{v}{V}} = \frac{p}{v} V \tag{153}$$

Since all changes in volume are conceived as being very small, if we assume a volume of gas V, to be subjected to a change in pressure dp, producing a corresponding change in volume dV, then

$$e = \frac{dp}{dV} V \tag{154}$$

It should be observed that the expression for the strain $\frac{dV}{V}$ denotes a *dilatation if positive*, and a *compression if negative.* The coefficient e, however, has reference simply to the numerical magnitude of the ratio, $\frac{dp}{\frac{dV}{V}}$, and is therefore independent of the sign.

Besides the elasticity of volume, solids have, as we have seen, elasticity of shape as well. If a solid be so distorted that its shape alone is changed, it is said to have undergone a *shear.* Thus if we conceive all the particles in a certain plane in a body to be fixed, and all the remaining particles to move in planes parallel to this plane, and by amounts proportional to their distances from this plane, the resulting distortion is called *a shear.* The stress arising from such a distortion is called a *shearing stress*, and the coefficient of rigidity, n, is the quotient obtained by dividing the shearing stress by the shearing strain.

59. Young's Modulus. If a load

$$F = mg$$

be suspended from a wire of uniform cross section, a, and length L, the wire lengthens by an amount l, and a strain is produced in the wire, equal to $\frac{l}{L}$. The stress due to this distortion resists the action of the stretching force, and equilibrium is established when the force due to the stress becomes equal to the load; or if

$$F = stress \times a = mg.$$

The coefficient of linear stretch is

$$M = \frac{stress}{strain} = \frac{\frac{mg}{a}}{\frac{l}{L}} = \frac{mgL}{al}. \tag{155}$$

This coefficient is called Young's modulus, and its unit is the dyne per cm². Under otherwise equal conditions the elongation of a wire will be smaller, the larger Young's modulus is for the substance of which the wire is made.

TABLE III.

COEFFICIENTS OF ELASTICITY

SUBSTANCE	YOUNG'S MODULUS	COEFF. OF VOLUME ELASTICITY
Brass	$11 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$	$10.5 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$
Steel	$22 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$	$18.5 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$
Cast Iron	$13.5 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$	$9.5 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$
Copper	$12 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$	$16.5 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}$

60. Three States of Matter. Matter presents itself in one of three states: *solid*, *liquid* or *gaseous*. These states are not separated by sharp lines of demarcation, but shade almost insen-

For experimental determination of Young's Modulus, see Manual, Exercises 22 and 23, sibly into each other, the particular state which the body assumes being dependent upon the temperature and pressure to which it is subjected.

A solid is a body which resists any force tending to change either its shape or its volume; in other words, it possesses elasticity both of shape and of bulk. It is characterized by small mobility of its molecules, and by the possession of surfaces of distinct outline on all sides.

A liquid is a body which has no elasticity of shape, or which offers no resistance to a shearing force; it is characterized by considerable mobility of its molecules, by a distinct, free, upper surface, usually of a meniscus shape when confined in a tube, and by the existence in that free surface of a specific tension, not found elsewhere in the body.

A gas is a body whose parts are held together only by the action of external force, and which offers resistance to the *decrease* but none to the *increase* of its volume. It is characterized by almost perfect mobility of its molecules. It has no upper surface and no surface tension, but tends to fill all available space.

Liquids and gases when subjected to shearing forces are unable to remain in equilibrium, and their parts move into new positions, or tend to *flow;* they are therefore classed together under the general term *fluids.* Fluids have perfect elasticity of volume but no elasticity of shape.

61. Intermediary Qualities. As has been said, the three states of matter shade gradually into each other, although the changes are more abrupt in some cases than in others. Thus while most substances are perfectly elastic under hydrostatic pressure, no substance has yet been found to be perfectly elastic under shearing forces producing finite strains. Owing to the varying behavior of substances under stress, a number of intermediary qualities are manifested, by virtue of which certain characteristics of the fluid state are seen to exist even in solids.

A body which breaks upon the sudden application of slight force is said to be *brittle.* Glass, gems and hardened steel are examples of brittle substances.

A *plastic* substance is one which has its form permanently altered by a force exceeding a certain value. Plastic solids under pressure behave as liquids, and flow with greater or less freedom. Thus cold lead under enormous pressure is made to flow through a die, producing lead pipe.

Plastic bodies may be beaten into various forms, rolled into sheets, or drawn into wire. This is exemplified in the case of copper and platinum and in still higher degree in the case of gold. Substances which may be beaten into thin sheets are called *malleable;* those that may be drawn into wire are said to be *ductile.* Certain substances are plastic at high temperatures and brittle at ordinary temperatures. Thus glass and quartz are extremely brittle at ordinary temperatures, yet when fused they may be drawn into threads of exceeding fineness. Quartz especially, when fused, may be drawn into fibers so fine as to be invisible to the naked eye, yet capable of supporting relatively large masses, and possessing almost entire freedom from elastic after effect.

Certain organic substances rival the metals in ductility. Thus sugar and pitch at certain temperatures may be drawn into threads so fine as to float on the air, while the spider's thread is formed in some cases of a thousand separate strands united into one.

62. Viscosity. While it is assumed that an ideal liquid would offer no resistance to a shearing stress, still it is to be noted that all known liquids do manifest a certain degree of resistance to the motion of their parts over each other. This *internal friction*, or *resistance to flow*, is called *viscosity.* A liquid that flows readily as alcohol or ether is termed a *limpid* liquid as opposed to a *viscous* liquid like honey or Canada balsam.

To illustrate the viscosity of liquids, support horizontally a glass disk 10 cm in diameter, by a stiff wire from its center, and lower it into a flat beaker containing several centimeters depth of glycerine, bringing the disk at least 5 cm below the surface of the liquid. Place small bits of paper or cork upon the glycerine and rotate the disk by twisting the supporting wire. The surface is soon seen to be in rotation, showing that the rotation of the disk sets up a shearing stress which is transmitted through the liquid to the surface layers and sets them in motion also. A smooth metal disk mounted horizontally upon a whirling table and set in rapid rotation will drag along with it the adjacent layers of air, as may be shown by lowering over it a similar disk suspended by a fine cord from its center. The viscosity of the air will soon suffice to set the upper disk in rotation even at a distance of one or two centimeters.

It thus appears that a column of glycerine or even a column of air is able, by virtue of the internal friction between its molecules, to transmit a shearing stress from one end of the column to the other. In the case of a perfectly rigid column such transmission would be instantaneous, while in the case of fluid columns there is a continual breaking down under the stress and the transmission is much slower and far less perfect.

This continued breaking down or yielding of the parts of a body under the action of a shearing stress is seen also in the case of bodies usually classed as solids. Thus shoemaker's wax, while brittle to a sudden blow, will creep slowly down an inclined plane under the action of gravity; long glass tubes, laid horizontally upon supports at the ends, sag down in the middle, due to their own weight; sandstone pillars under enormous weights have slowly bent without breaking. All these are examples of viscous substances.

63. Coefficient of Viscosity. Poiseuille's Law. Internal friction likewise determines the velocity with which a liquid under pressure will flow through a tube of narrow bore. Since the layer of liquid in contact with the tube remains practically at rest, the velocity of flow increases from without inward, becoming a maximum at the center. With a definite difference of pressure between the two ends of the tube, the liquid assumes a steady state of flow, and the moving mass may be considered as made up of a number of concentric liquid tubes slipping over each other under pressure and retarded by molecular friction. With a difference of pressure p, between the two ends of the tube, the quantity Q, discharged through a tube of length l, and radius r, in time t, is given by the equation

$$Q = \frac{\pi p r^4}{8 l \eta} \cdot t \qquad (156)$$

where η is called the *coefficient of viscosity.*

This relation, first determined experimentally by Poiseuille, a French physician, and subsequently derived theoretically by others, is known as ***Poiseuille's law.*** It has been found to hold very accurately for small velocities of flow, but fails when the velocity exceeds a certain value depending upon the size of the tube and the nature of the liquid. Poiseuille's law is of importance in questions relating to the capillary circulation of the blood.

The coefficient of viscosity η is defined as numerically equal to the tangential force exerted upon unit area of either of two horizontal planes placed at unit distance apart, in the viscous substance, one of the planes being fixed and the other moving with unit velocity. This coefficient may be measured in several other ways, as by determining the resistance encountered by a solid in moving through the viscous liquid.

The value of η varies greatly for different substances. For water at 15° C its value is 0.0115; for glycerine it is about 300 times as much; for ether one third as much; and for air one fifty-fifth as much. The viscosity of liquids usually diminishes with increase of temperature. Hot water is less viscous than cold; sirups and saline solutions are limpid when hot and viscous when cold. In gases, on the other hand, the viscosity *increases* as the temperature increases.

MECHANICS OF FLUIDS

CHAPTER VII

FLUIDS AT REST

64. Fluid Pressure. A perfect fluid is a body which would offer no resistance to a shearing stress. This, as we have seen, is a purely ideal case, but if we consider the distortions in a fluid mass as vanishingly small, even the slight resistance of the fluid to change of form arising from viscosity becomes zero, and we find that *fluids at rest behave as perfect fluids*. Since in a perfect fluid there is no viscosity, and hence no statical friction, it follows that the *pressure* exerted by the fluid on the containing vessel must all be *normal* to the *surface* of the vessel, because any pressure other than a normal pressure could be resolved into a normal and a tangential component. The normal component would produce pressure on the side of the vessel; the tangential component would produce motion in the fluid unless opposed by statical friction. But there is no statical friction, and since the fluid *is at rest*, there can be *no tangential component*. Therefore the fluid pressure on the side of a vessel is normal to the side of the vessel.

Again, it follows from the absence of statical friction in a fluid that any pressure exerted upon any surface of a confined fluid must be transmitted undiminished to every area of surface in contact with the fluid. For consider two cylinders connected with a closed vessel (Fig. 31), each cylinder being stopped by a piston of area a, and the whole filled with a fluid. If now one piston be forced in through a distance b, then the second piston must be driven out an equal distance, if no compression of the fluid is to ensue. The work done by the first piston is

Fb, where *F* is the force applied. Since no work is done in compressing the liquid, the work done on the second piston must be *F′b*. Whence

$$Fb = F'b \qquad (157)$$

or

$$F = F' \qquad (158)$$

Even if the pistons have unequal areas, *a* and *a′*, the pressure is still transmitted undiminished, since in this case we have

$$Pab = P'a'b' \qquad (159)$$

and *ab*, the volume of liquid displaced on one side, is equal to *a′b′*, the volume displaced on the other side, if no compression takes place. Whence

$$P = P' \qquad (160)$$

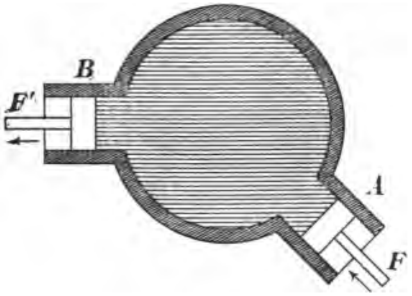

Fig. 31.

65. Pressure at Any Point in a Fluid. Since pressure in a fluid is transmitted undiminished in all directions, it follows that the pressure at any point is the same in all directions. For suppose a body of liquid to be in equilibrium, and conceive a cylinder of the fluid of very small diameter to become solid without change of density. Then unless the pressures upon the ends of the cylinder are equal and oppositely directed, motion will ensue. But the liquid is at rest; hence there is no difference in the pressures upon the two ends. Now since the cylinder may be rotated in a horizontal plane, it follows that at any point in a liquid the horizontal pressure is the same in all directions, and since the diameter of the cylinder may be indefinitely diminished, the vertical pressures must be equal for the same reason.

This may be illustrated experimentally by the apparatus shown in Fig. 32. Bend three glass tubes of about 5 mm diameter into the forms of tubes *I*, *II*, *III*. Fill the lower parts of the three tubes to the same height with mercury and immerse in a vessel of water, keeping the mouths of the three tubes in a horizontal line. The difference in level of the mercury in the two arms of each tube is the same; thus showing that in that horizontal plane in the liquid, the upward, downward and lateral pressures are equal. On lowering the tubes to the bottom this difference in level increases; thus showing that the pressure increases as the depth increases.

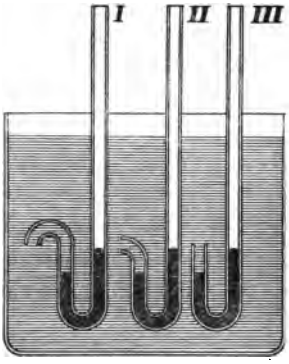

FIG. 32.

66. Free Surface of a Liquid at Rest, Horizontal. It may readily be shown that the free surface of a liquid at rest is horizontal. Let A (Fig. 33) be a particle of liquid of mass m, on a surface that is *not* horizontal. Then the force of gravity on the particle is mg in the direction AB. This can be resolved into a normal component AC, and a tangential component AD. The normal component simply produces pressure on the interior of the liquid and may be neglected. The tangential component AD is free to produce motion. Its value is

$$AD = mg \cos BAD \tag{161}$$

But by hypothesis the liquid is at rest; this cannot be true unless AD is zero. But since mg cannot be zero, $\cos BAD$ must be zero; that is, BAD is 90°, or the surface of a liquid at rest must be horizontal.

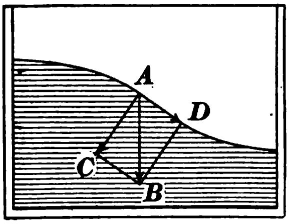

FIG. 33.

67. Pressure on an Immersed Surface due to the Weight of a Liquid. Consider a surface of area A immersed in a liquid of density d. We shall first find the force upon this surface due to the weight of the liquid. Let the area be subdivided into a large number of very small areas, $a_1, a_2, a_3 \ldots\ldots a_n$; then

$$A = \Sigma a \tag{162}$$

If we take one such small area as a (Fig. 34) at a depth h from the surface, then the mass of liquid pressing upon this surface is the volume ah, multiplied by the density, or ahd, and the *force* f, on this small surface is $ahdg$. The total force on the surface A is therefore

$$\begin{aligned} F &= a_1h_1dg + a_2h_2dg + a_3h_3dg + \ldots\ldots a_nh_ndg \\ &= (a_1h_1 + a_2h_2 + a_3h_3 + \ldots\ldots\ldots\ldots a_nh_n)dg \qquad (163) \end{aligned}$$

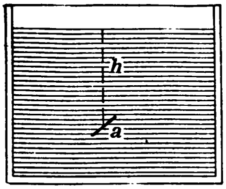

FIG. 34.

Let H be the distance from the center of area of the surface pressed upon, to the surface of the liquid. Then by equation (95) we have

$$H = \frac{\Sigma ah}{\Sigma a}$$

or

$$F = AHdg \qquad (164)$$

This is a general expression for the force due to the weight of a liquid upon any surface immersed in it, in any position whatever. The expression shows also that the force upon an immersed surface varies directly as the area of the surface, as the depth of the center of area below the surface of the liquid, as the density of the liquid, and as the acceleration due to gravity.

Equation (164) also gives an expression for the average pressure exerted by a liquid upon an immersed surface. For

$$P = \frac{F}{A} = Hdg \qquad (165)$$

while the pressure p, at any point in the liquid, is obviously

$$p = \frac{f}{a} = hdg \qquad (166)$$

68. Principle of Archimedes. *A solid immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced.* This principle is readily shown to be true by considering a vessel filled with liquid which is in equilibrium. This condition of affairs will not be disturbed if we suppose a certain part of the liquid to become solid without change of density. The forces acting on the solidified portion are its weight acting downward through its center of gravity, and the force exerted by the liquid upon its outer surface. Since this equilibrates the weight of the solidified portion, it follows that the resultant of all the forces due to the fluid must be directed upward through the center of gravity of the solidified portion, and must be equal to the weight of that portion. Consequently, the buoyant force exerted upon any body immersed in a fluid (Fig. 35) is equal to the weight of the fluid displaced by the body, and acts through the center of gravity of the fluid displaced.

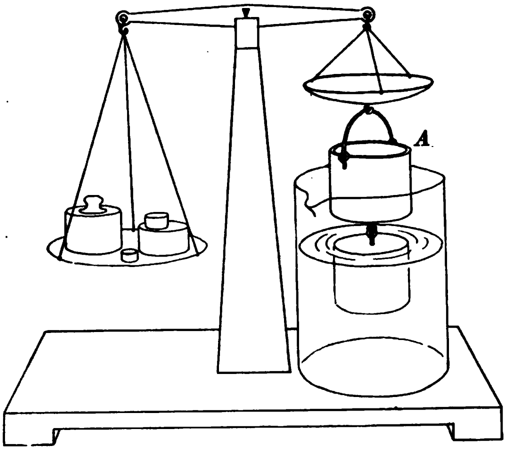

Fig. 35.

A result of this fact is that any body weighed in a fluid weighs less than if weighed in air, by an amount equal to the weight of the fluid displaced. This seeming loss of weight is due to the buoyant force of the fluid in which the body is immersed. Hence to find the volume of any body of irregular outline it is only necessary to weigh it first in air and then in distilled water at 4° C. The apparent loss of weight is the weight of a volume of water, equal to the volume of the body, and if the weight be expressed in grams, the volume is at once obtained in cubic centimeters. In this way the number of cubic centimeters of iron in a handful of carpet tacks may be determined.

In accordance with the principle of Archimedes, it is necessary in accurate weighing to correct for the buoyancy of the air. This precaution is unnecessary in case the object weighed and the weights used have the same density; it is the more necessary, the smaller the density of the body weighed.

If we immerse a body in a fluid whose density is greater than that of the body, the body will displace more than its own weight of the fluid and will therefore rise until the weight of the fluid displaced just equals its own weight, when it floats. Hence *a floating body displaces its own weight of the liquid in which it floats.*

Finally, if we call m the mass of the body,

m_1 its apparent mass when weighed in the fluid,
d the density of the body,
d_1 the density of the fluid,
v the volume of the body, and if we set the apparent loss

of weight equal to the buoyant force of the fluid, we have

$$mg - m_1g = vd_1g \tag{167}$$

69. Density and Specific Gravity. *The density of a body is its mass per unit volume.*[1] The specific gravity of a body is the ratio between the density of that body and the density of some substance assumed as a standard. For liquids and solids the standard is distilled water at a temperature of 4° C. For gases hydrogen is usually assumed as the standard. If the mass and volume of a body be known, its density is given by the relation

$$d = \frac{m}{v}$$

Ordinarily the volume of a body is found in accordance with the principle of Archimedes, by finding the mass m, of the body in air and its apparent mass m_1, in some fluid of known density,

[1] For experimental determination of density – see Manual, Exercises 16–18,

d_1. The difference is the mass in grams of the fluid displaced by the body. The density of the body will then be given by the expression obtained in the previous article, on substituting for v its value m/d, or

$$d = \frac{m}{m - m_1} \cdot d_1 \tag{168}$$

In case the fluid is distilled water at 4° C, d_1 is taken as unity. If the weighing be done at some temperature other than 4° C, or the fluid be other than distilled water, then d_1 represents the density of the fluid used, *at the temperature at which the weighing was done.*

For bodies lighter than water, a sinker with a sharp point is hung from the left-hand pan, immersed in the standard fluid and counterpoised by small shot or fine sand. The body whose density is sought is placed in the left-hand pan and its mass m determined as usual. It is then fastened to the sinker by sticking it upon the sharp point immersed in the fluid, and equilibrium restored, either by adding masses to the left-hand pan, or removing them from the right-hand pan. The apparent change in mass represents the mass of the liquid displaced, or $m - m_1$. Whence, as before,

$$d = \frac{m}{m - m_1} \cdot d_1$$

For finding the density of liquids, the most accurate method is by means of the pyknometer, or "specific gravity bottle." This is a small flask fitted with a ground glass stopper which is perforated throughout its length for the escape of superfluous fluid on filling the flask. The flask is first cleaned and dried and weighed empty. It is then filled with the liquid and weighed, and finally filled with the standard and weighed. These two weights, less the weight of the empty flask, give at once the masses of equal volumes of the substance and of the standard; whence we have

$$d = \frac{m}{v}$$

Other methods are frequently employed for determining the density of liquids, especially when great accuracy is not required.

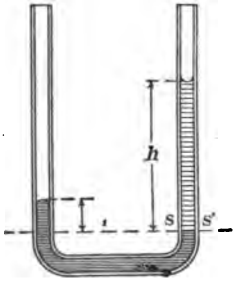

FIG. 36.

70. Liquids in Communicating Tubes. Consider a bent tube (Fig. 36) containing two liquids which do not react chemically upon each other. When the system has come to rest, it will be found that the less dense liquid stands at a height h above a horizontal line through the junction ss' of the two liquids. The pressure exerted by this column is evidently balanced by the pressure due to the denser liquid whose height above the same surface is h_1. Let d and d_1 be the respective densities of the two liquids. The equation of equilibrium is then

$$h_1 d_1 g = hdg \tag{169}$$

whence

$$\frac{d_1}{d} = \frac{h}{h_1} \tag{170}$$

or the heights of the two liquids above their common plane of separation vary inversely as their densities.

In case the liquids react chemically upon each other, the device shown in Fig. 37 may be used. The bent tube is inverted and the ends placed in cups containing the liquids of densities d and d_1. Through a short tube at the top the air may be partly removed from the tubes, producing a difference of pressure P between the air inside and outside the tube. This difference is balanced in each case

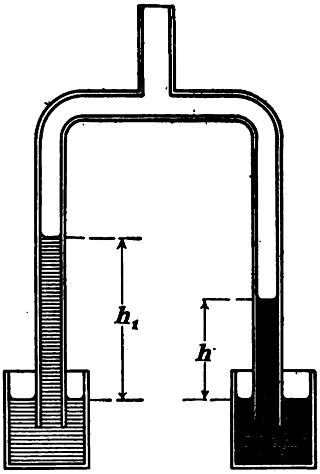

FIG. 37.

1 line short

by the rise of the liquid in the two tubes, and we have for equilibrium

$$h_1 d_1 g = hdg$$

as before.

If the density of one of the liquids be known, the density of the other may be computed at once.

TABLE IV

Densities of Various Bodies

Alcohol at 20° C	0.789	Iron, wrought	7.86
Aluminum	2.58	Lead	11.3
Brass (about)	8.5	Mercury at 0° C	13.595
Brick	2.1	Nickel	8.9
Copper	8.92	Oak	0.8
Cork	0.24	Paraffin	0.9
Diamond	3.52	Pine	0.5
Glass, common	2.6	Platinum	21.5
Glass, heavy flint	3.7	Quartz	2.65
Gold	19.3	Silver	10.53
Ice at 0° C	0.916	Tin	7.29
Iron, cast	7.4	Zinc	7.15

71. The Barometer. A barometer is an instrument for measuring the pressure of the atmosphere. It is made by taking a stout glass tube (Fig. 38) about 80 cm long and closed at one end, filling it with mercury, and inverting it with the open end under the surface of mercury in a shallow dish. When the system has come to rest, the mercury in the tube stands about 76 cm above the level of the mercury in the dish, if the experiment be performed at sea level, in latitude 45°.

Now since pressure in fluids is transmitted undiminished in all directions, it follows that the weight per unit area of this column of mercury, 76 cm in height, must be balanced by the downward pressure of the air upon the surface of the mercury in the dish. If for any reason the atmospheric pressure change, the corresponding difference in the height of the mercury in the tube enables us to measure this change.

If the barometer be carried up the side of a mountain, or down into a mine, the elevation above, or the depression below, the sea level may be roughly determined from the difference in the barometric readings. At sea level a change of 11 m in level produces a change of 1 mm in the height of the barometer column. This rate of change is not constant, but diminishes as the elevation increases. Correction, however, should be made for differences in temperature at the various heights.

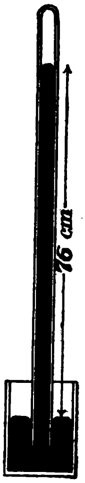

FIG. 38.

The pressure of the atmosphere as measured by the barometer is readily computed. Assume the tube (Fig. 38) to be of unit cross section and g to be $980 \frac{\text{cm}}{\text{sec}^2}$. Then the volume of mercury supported is 76 cc; the mass is 76 × 13.596 = 1033.296 g, and the force is 1033.296 × 980 = 1,012,630 dynes. Hence the atmospheric pressure is 1,012,630 dynes per square centimeter. This pressure is called the ***pressure of one atmosphere.***[1] In English units the pressure of one atmosphere is approximately equal to a weight of 14.7 lb per square inch.

It should be noted that the barometric height is constantly changing, and that the standard height of 76 cm or 30 in is the average height for places at the *level of the sea, latitude* 45° *and at the temperature of* 0° *C.* Ann Arbor is 882 ft above sea level, and the mean barometric reading for the year 1901 was 29.03 in.

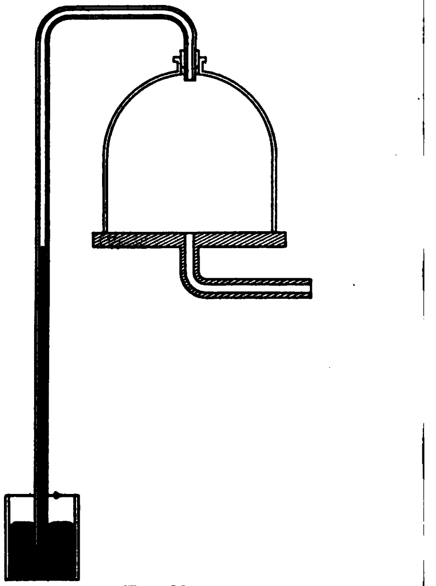

FIG. 39.

72. Manometers. Let a tube be attached to a bell jar and the jar be placed upon the plate of an air pump with the lower end of the tube dipping into a cup of mercury, as shown in Fig. 39. On withdrawing the air from the bell jar

[1] For directions for [illegible]
in Manual, [illegible]

the mercury rises in the tube, thus furnishing a measure of the exhaustion as the pumping proceeds. Such an arrangement is called *a manometer.* For measuring slight differences in gaseous pressure, the second form shown in Fig. 40 is used, where the total pressure upon the gas in the horizontal arm is one atmosphere, plus the mercury column *AB.* A more sensitive form of this instrument is secured by substituting some light oil in place of mercury.

For the measurement of pressures amounting to several atmospheres the *compression manometer* (Fig. 41) is a convenient and compact form.

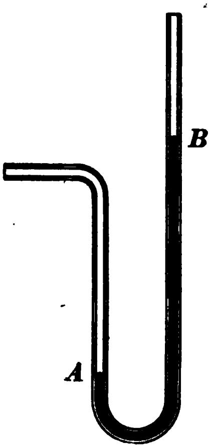

FIG. 40.

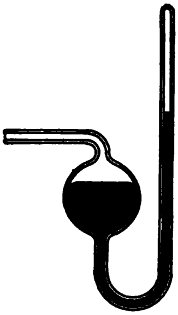

FIG. 41.

73. Pumps. In the action of the ordinary lifting pump (Fig. 42) we have an application of the pressure of the atmosphere. This pump consists of a piston *D*, working air tight in a cylinder *HK*, to which is attached a pipe of smaller diameter, closed by the valve *A*. Suppose the pump to be inserted in the water at *L*, and the piston to be at the bottom of the cylinder. On raising the piston the weight of the air closes the valves *B* and *C* in the piston, and the air in the small pipe expands, raises the valve *A*, and passes in to fill the vacant space in the cylinder. The pressure of the air in the pipe is diminished and the hydrostatic pressure due to the weight of the atmosphere drives the water up into the small pipe. On the downward stroke

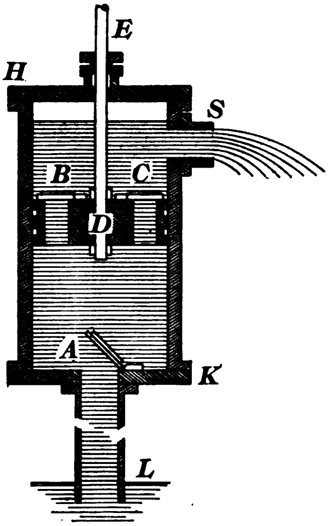

FIG. 42.

the air in the cylinder closes the valve *A*, and escapes through the valves *B* and *C* into the outer air. By the second and succeeding strokes of the piston, the air below is still further rarefied, until the water rises above the valve *A*, passes above the piston through the valves *B* and *C* on the next downward stroke, and is finally lifted to *S*, where it flows out. Since the density of mercury is 13.6 times that of water, it follows that the pressure of the atmosphere will support a column of water 13.6 times the barometric height; or 30 in × 13.6 = 34 ft. This would represent the maximum effect to be obtained from the pressure of the air; as a matter of fact an ordinary pump will not raise water much more than 26 ft.

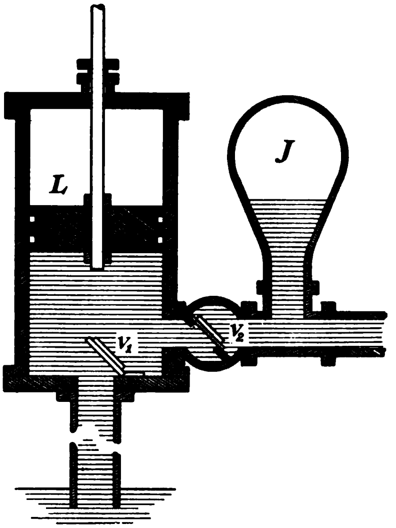

FIG. 43.

The *force pump* (Fig. 43) is provided with a solid piston *L*, and a second pipe is attached to the cylinder above the valve V_1. This pipe leads to an air chamber *J*, into which the water is driven on the down stroke of the piston, and retained there by a valve V_2. An outlet pipe fitted with a nozzle delivers the water driven out of the air chamber by the force of the compressed air; in other respects the working of the pump is similar to that of the lifting pump. The advantage of the air chamber is that it produces a steady stream from the nozzle instead of an intermittent one.

74. The Siphon. The siphon (Fig. 44) is an instrument for transferring liquids from one vessel to another at a lower level. It consists of a bent tube, with arms of unequal length filled with liquid and inverted with the shorter arm in the vessel from which the liquid is to be transferred. The distances

a and b, from the surfaces of the liquid in the vessels to the highest point of the bend of the tube, represent the two columns of liquid in motion.

Let h be the height of the column of liquid which the atmospheric pressure will support at the time and place of the experiment, and d the density of the liquid. Then the pressure to the right, on a plane through c, the highest point in the tube, is

$$hdg - adg \qquad (171)$$

and the pressure to the left is

$$hdg - bdg \qquad (172)$$

or the difference in pressure tending to produce flow is

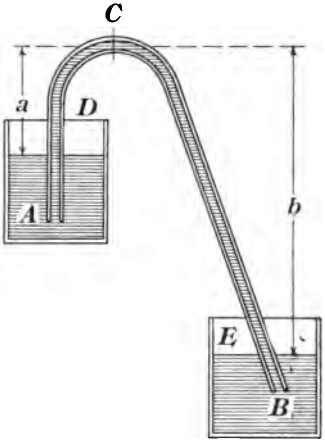

Fig. 44.

$$(hdg - adg) - (hdg - bdg) = (b - a)dg \qquad (173)$$

If a equal b, the flow ceases. If a be greater than b, the liquid flows in the opposite direction.

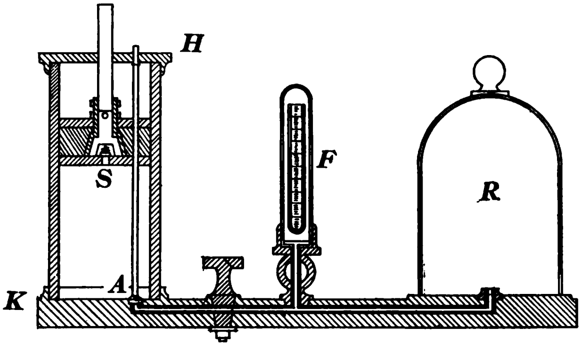

Fig. 45.

75. The Air Pump. The action of the air pump is essentially the same as that of the common lifting pump.

In addition to the parts described (Art. 73) the air pump is fitted with an automatic valve A (Fig. 45), which opens at the up stroke of the piston and closes on each down stroke. This is effected by making the valve *A*, in the form of a conical plug attached to a rod passing through the piston, which slides upon the rod with a small degree of friction. By this means the valve *A* is opened to admit air from the receiver, which by reason of its diminished density would be unable to lift the valve by its expansive force. A manometer *F* shows the degree of exhaustion in the receiver as the action of the pump proceeds. Owing to unavoidable wear and consequent leakage in the pump itself, it is impossible to secure a high degree of exhaustion with a mechanical air pump of the ordinary type.

For work requiring a high degree of exhaustion, as in the production of Geissler or Roentgen tubes, a mercury air pump is employed. Such pumps are usually made almost entirely of glass, and the piston is a column of mercury, which, on rising, fills a large bulb, expelling the air through a barometric manometer. On descending, the air from the vessel to be exhausted enters the bulb through a glass-mercury valve, and expanding, fills it at a much diminished pressure, only to be forced out as before by the liquid piston. By the use of such pumps pressures of one millionth of an atmosphere are readily attainable. For such purposes a variety of forms of the mercury pump have been devised, the idea in each case being to secure rapid automatic action.

76. Weight and Density of Air. Galileo satisfied himself that air had weight by weighing a glass globe first filled with air at ordinary pressure and then filled with air under high pressure. The experiment is ordinarily performed to-day by weighing a stout glass balloon when filled with air and then when the air has been exhausted. A liter of ordinary air, containing 0.04 % of carbon dioxide, under the standard conditions of 0° C. and 76 cm pressure, weighs 1.293 g. Hence the density of air is 0.001293 g per cubic centimeter.

In the use of a delicate balance it is frequently necessary to correct the weighings for the buoyancy of the air in which the substance is weighed, in accordance with the principle of Archimedes. Thus let w be the weight of the body weighed,

w' the weight of the brass weights, and a and a' the buoyant force of the air upon the body and weights respectively. Then for equilibrium

$$w - a = w' - a'$$

or

$$w = w' + a - a' \tag{174}$$

Thus the correction to be added to the weight of 25 g of cork, whose density is 0.25 g per cubic centimeter is obtained as follows:

$$a = \frac{25}{0.25} \times 0.001293 = 0.1293 \text{ g}$$

$$a' = \frac{25}{8.5} \times 0.001293 = 0.0038 \text{ g}$$

Or the correction to be added is 125 mg.

77. Boyle's Law. Mention has already been made of the work necessary to compress a gas in a cylinder, and of the resistance offered by a gas to any force tending to *decrease* its volume. The relation between the volume of any mass of gas and the pressure exerted by the gas upon the walls of the containing vessel was first stated by *Robert Boyle* in 1662, although its far-reaching importance was more fully realized by *Mariotte*, a French physicist, who rediscovered the law, independently, in 1676.

Boyle's law states that, *at a constant temperature, the volume of a body of gas varies inversely as the pressure to which it is subjected.* 1) Thus if p_0 and v_0 be the initial pressure and volume of a mass of gas, and p and v the final pressure and volume, then

$$pv = p_0 v_0 = \textit{constant} \tag{175}$$

or *for a constant temperature, the product of the pressure and the volume is a constant.* Since the density of any body varies inversely as the volume, the law may be stated in another way, viz.:

$$\frac{p}{d} = \frac{p_0}{d_0} = \textit{constant} \tag{176}$$

or *the pressures are directly proportional to the densities.*

1) For experimental verification of Boyle's law see manual, Exercise 19.

Careful experiments have shown that for high pressures or low temperatures, Boyle's law is only approximately true. Thus it has been shown that all gases may be liquefied by the application of pressure and the reduction of the temperature. As the gas approaches the point of liquefaction, the variation from the law is most noticeable. Gases which can be liquefied by pressure at ordinary temperatures, such as chlorine, sulphur dioxide, and carbon dioxide, can hardly be said to obey the law at all for small differences in pressure, while for gases like nitrogen, oxygen and hydrogen, the law is very nearly true at ordinary temperatures.

An ideal or perfect gas would obey Boyle's law at all temperatures, and in general it may be said that the farther removed any gas is from its point of liquefaction, the more nearly it behaves as a perfect gas. This departure from the law by all gases when near the point of liquefaction has been attributed to the action of two causes: (*a*) *The attraction of the molecules of the gas for each other.* This would increase as the compression increases and therefore assist the compression. (*b*) *The size of the molecules*, which would tend to retard the compression of the gas. More complete formulae for the behavior of a gas under varying pressure and temperature have been proposed by Van der Waals (see Art. 219), Clausius and others.

CHAPTER VIII

FLUIDS IN MOTION

78. Velocity of Efflux. When a small opening is made in the side of a vessel filled with liquid, at a distance h below the surface, the liquid flows out with a definite velocity v, where

$$v^2 = 2\,gh$$

This formula for the velocity of a liquid issuing under pressure due to a head h is known as *Torricelli's theorem.* It may be demonstrated as follows: Suppose a particle of liquid of mass m to be situated in the surface of the liquid. Its potential energy with respect to the orifice is mgh. In passing from the surface of the liquid to the orifice it has fallen a distance h, and, if we neglect the viscosity of the liquid, its kinetic energy on emerging from the orifice must equal its potential energy at the beginning, or

$$\frac{mv^2}{2} = mgh \tag{177}$$

whence, as before,

$$v^2 = 2\,gh$$

If a be the area of the orifice, then V, the volume of liquid discharged in time t, would be

$$V = avt \tag{178}$$

In practice this rate of discharge is never reached. If the opening be a simple orifice in the side of the vessel, without mouthpiece of any sort, the volume of liquid discharged is about 0.62 of the theoretical value. This difference is due to the convergence of the lines of flow, producing a contraction of the jet just outside the orifice, whereby the actual cross section of the stream at this point is much reduced. By using a short cylindrical mouthpiece, of length two or three times its diameter, and set flush with the inside of the vessel, the flow may be made 0.82 of the theoretical value; while by so shaping the mouthpiece as to conform most closely to the form of the contracted jet, a velocity but little short of $\sqrt{2gh}$ is attained.

79. Velocity of Effusion for Gases. Consider a vessel filled with gas, and having an orifice at one side of cross section a, from which the gas escapes with a velocity v. Let the density of the gas be d g/cm³, and the pressure be p dynes/cm², above that of the air. The volume of gas delivered in t sec will then be avt cc, and the mass $avtd$ grams. The kinetic energy of the escaping gas will be $\frac{av^3td}{2}$ ergs. If the gas be allowed to do work by pushing back the surrounding air, then the work done on the air must equal the kinetic energy of the gas, if we assume the outflow to take place so slowly that the gas is not cooled below the temperature of the surrounding air. The work will then be the product of the force ap and the distance vt, or

$$\frac{av^3td}{2} = apvt \tag{179}$$

whence

$$v = \sqrt{\frac{2p}{d}} \tag{180}$$

or the velocity with which a gas effuses through an opening varies directly as the square root of the difference of pressure on the two sides of the opening and inversely as the square root of the density of the gas.

From this it follows that two gases effuse through the same opening, under the same difference of pressure, with velocities inversely as the square roots of their respective densities. Bunsen has utilized this principle to compare the densities of gases by observing the time required for the same volume of the various gases to effuse through the same opening under the same difference of pressure.

If the pressure p be expressed in terms of a column of the gas extending h cm above the orifice, equation (180) then becomes

$$v = \sqrt{\frac{2\,ghd}{d}} = \sqrt{2\,gh} \qquad (181)$$

80. Flow of Liquids through Tubes. If instead of a metal tube, an elastic tube of rubber be attached to the orifice of the discharging vessel, the efflux is the same as that by a rigid tube of the diameter assumed by the elastic tube, *so long as the flow is uniform.* If, however, the flow by any means be made to assume an intermittent or pulsating character, the discharge from the elastic tube is notably larger than from the rigid tube. The nature of the discharge is also modified, in that the stream from the rigid tube reproduces faithfully every feature of the pulsating impulse, while the elastic tube rapidly smooths out the inequalities of pressure, so that in an elastic tube of sufficient length the pulsation disappears entirely. This fact is of importance in explanation of the flow of the blood through the arteries, the coats of which are extremely elastic.

Again, the flow of liquids through tubes is much retarded on account of friction, not only among the particles of the liquid but between the liquid and the walls of the tube. This latter friction is much the more important of the two, and increases rapidly with the roughness of the walls of the tube. The flow of a river is greatest at the center of the stream, and at the surface of the water, where the effect of friction from the bed and banks is as small as possible.

In a vertical tube the liquid column breaks into a series of liquid masses fitting the tube more or less perfectly. These masses acquire an increasing velocity in their descent and act as liquid pistons fitting the interior of the tube. A partial vacuum results and the water is forced into the pipe more rapidly on this account. The effect of this exhaustion causes the noisy draught with which the last portions of water leave a wash basin or a bath tub, where the waste pipe is long and free. This action of vertical waste pipes is also liable to draw out the water from the siphon traps, and leave the way open for the entrance of poisonous sewer gas. For this reason all waste pipes from basins, closets, bath tubs, etc., are now required to have separate vent to the outside air.

This reduction of pressure by liquids in vertical tubes is utilized in the Bunsen air pump, where a vertical column of water of more than 34 ft is made to exhaust the air from a receiver; the limit of the exhaustion being of course the pressure equal to the vapor tension of water at the existing temperature. In the Sprengel pump the liquid is mercury. This has two advantages: it requires a vertical column but 30 in long, and the vapor tension of mercury is practically negligible.

81. Flow in Pipes of Variable Section. In a tube of variable cross section the flow of liquids presents some interesting features. In Fig. 46 the variation of the pressure exerted by the fluid upon the walls of the tube is shown by the manometer tubes. It is thus seen that in a tube of variable cross section running full of liquid, the pressure is greatest in the widest part of the tube and least in the narrowest part. This somewhat surprising result is easily explained by considering the conditions of flow in the different parts of the tube. It is readily seen that with steady flow the velocity is greatest where the cross section of the stream is least, and *vice versa*. The liquid in passing from a wider to a narrower part of the tube must, therefore, undergo an acceleration, since for steady flow the same volume must pass any section in the pipe in the same time. To produce this acceleration the pressure on any section must be greater from *behind;* similarly, in passing from the narrower to the wider part of the tube, the velocity *decreases*, or the acceleration is *negative;* hence the pressure is greater from *before* than from *behind*, so that the pressure is greatest

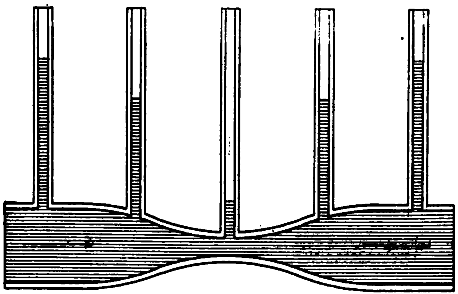

FIG. 46.

in the widest part of the tube and least in the narrowest part.

82. Jet Pump. The reduction of pressure within a contracted stream has been applied in the construction of many useful pieces of apparatus. In Fig. 47 is seen a common form of aspirator or jet pump as used in the laboratory. Water entering through the tube *E*, under hydrant pressure, passes through the constricted inner tube at *A*, and flows out at *D*. Owing to the small cross section of the stream at *A*, the velocity is very great, and the pressure is so much reduced that the air from the tube *B* is drawn along through the constricted portion in a torrent of small bubbles and carried down the tube *D*. With a well-constructed pump of this kind, a vacuum of about 5 cm of mercury may be obtained, with water from the city water mains. Obviously the pump will work equally well if the water enters at *B*, and the air through *E*.

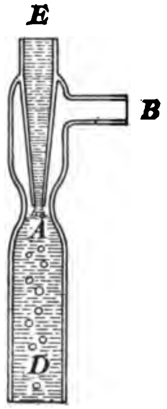

FIG. 47.

This pump has been adapted to numerous uses. The filter pump of the laboratory, the atomizer for spraying of perfumes or medicines, the injector in steam boilers, and the forced draught as used on locomotives are all different forms of this apparatus.

Problems

1. Find the weight on the bottom of a tank 10 ft square and 5 ft deep, when full of water. Find the force on one side of the same tank.

2. A triangular plate is immersed vertically in water, with the vertex in the surface of the water and the base horizontal. The height and base of the triangle are each 50 cm. Find the force on the face of the plate.

3. What is the density of a body whose mass is 678 g, if it weigh 235 g when immersed in a fluid whose density is 1.94 g per cubic centimeter?

4. A piece of wood of density 0.6 g per cubic centimeter floats on water. The volume of the wood is 40 cc. What is the volume of the water displaced?

5. A body having a density of 2.35 g per cubic centimeter weighs 624 g when immersed in a liquid whose density is 0.827 g per cubic centimeter. What is the mass of the body?

6. If the density of ice be 0.9179 g per cubic centimeter, and of sea water be 1.025 g per cubic centimeter, what portion of an iceberg is above water?

7. A piece of silver and a piece of gold ore are suspended from the ends of a balance beam having equal arms. The balance is in equilibrium when the silver is immersed in alcohol (sp. gr. 0.85), and the gold in nitric acid (sp. gr. 1.5). If the specific gravities of the gold and silver be 19.3 and 10.5 respectively, find the ratio of their masses.

8. A U-tube is partly filled with water. How many centimeters of oil having a density of 0.79 g per cubic centimeter must be added in order to raise the water in one leg 4.5 cm above its first level?

9. Twenty-four cubic centimeters of gas at a pressure of 71 cm of mercury would have what volume under a pressure of 76 cm?

10. A liter of air under normal conditions of temperature and pressure weighs 1.293 g. Find the weight of the air in a liter flask when the barometer stands at 72 cm, the temperature being 0° C.

11. To what depth must a diving bell 6 ft high be immersed in order that the water may rise 4 ft within it?

12. A glass tube used for sounding is 38.1 cm long and open at the lower end. The inside is covered with a soluble pigment, and the tube lowered to the bottom, in sea water, density 1.03 g per cubic centimeter. On raising it to the surface it is found that the water had entered the tube to a depth of 23.6 cm. Find the depth of the sea water.

13. A vessel filled with water has a circular orifice 6 cm in diameter, 298 cm vertically below the surface of the liquid. If the water be maintained at its initial depth, by supply from without, calculate the theoretical discharge per minute.

14. A picture of mass 4 kilos is suspended in the ordinary way by a cord fastened to two hooks and passing over a smooth nail. The hooks are 45 cm apart and the cord is 120 cm long. Find the stretching force in the cord.

MOLECULAR MECHANICS

CHAPTER IX

SURFACE PHENOMENA

83. Molecular Forces. By molecular forces are meant all those forces whose range of action is confined to insensible distances; that is, to distances comparable to the spaces between the individual molecules of a solid or a liquid. Under this head belong the forces of adhesion, cohesion, friction, viscosity, elasticity, capillarity, and surface tension; and although certain of these have been mentioned in previous topics, it seems proper to classify them here under the general term *molecular forces.*

The magnitude and importance of these forces are apt to be underestimated. It is owing to the action of molecular forces that any solid body is not only kept from falling to pieces of its own weight, but is able to resist the application of enormous stress as well. A clean glass tube cautiously lowered to the surface of clean water exhibits no attraction for the water, and causes no change in its upper surface so long as there exists any appreciable distance between them. But if the glass tube touch the surface, the water promptly runs up into the tube and clings to the outside, so that when the tube is withdrawn, a drop of water hangs to the lower end and the force of gravitation is unable to pull it off. Clearly we have here to do with forces, in comparison with which the force of gravitation is weak and insignificant.

The attraction between *unlike molecules*, as between those of water and glass, is called *adhesion;* that between *like molecules*, as between those of water and water, is called *cohesion.* These are in reality only different names for the same thing, viz.: molecular attraction, and it is to be noted that this attraction is exhibited only so long as the substances are *in contact;* that is, it acts through insensible or molecular distances. From a study of the behavior of gases we are led to believe that elasticity is due to molecular forces and molecular motions, and that the same is equally true of capillarity, surface tension and viscosity.

84. Adhesion and Cohesion. If two smooth, freshly cut surfaces of lead be firmly pressed together with a slight twisting motion, they cling together with considerable force, but having once been pulled apart they can be made to stick again only by application of sufficient force to bring the surfaces into close contact. A pair of glass plates, if highly polished, plane and free from dust, may be pressed together so firmly that it is impossible to separate them without rupture. That this is not due to the pressure of the air is shown by the fact that the plates cling together more firmly in a vacuum than in the open air, owing to the removal of the air film between the plates.

The adhesive action of glue, cement, mucilage and such substances renders it possible to unite two bodies so firmly that they break before separating. Dissimilar substances are united with difficulty, owing to their different rates of expansion or contraction when heated or cooled. Thus it is impossible to seal an iron or copper wire into a glass tube, since the metal and the glass have different rates of expansion; platinum, on the other hand, and certain alloys of nickel and iron may be sealed into glass, since they expand and contract at the same rate as the glass. For the same reason different kinds of glass frequently cannot be made to hold when sealed together.

Gases adhere to solids with great tenacity. It is almost impossible to free a glass tube from the adhering film of air, and consequently in the making of barometers, thermometers, and vacuum tubes of any description the air film is removed from the inner surface of the glass only by repeated heating and pumping.

The cohesive force of water is illustrated by hanging a clean, smooth glass plate to one arm of a balance so that it is horizontal, and bringing under it a jar of clean water. On touching the under side of the plate to the water, taking care to avoid air bubbles, it will be found necessary to add a relatively large weight to the opposite scale pan in order to pull the disk squarely away from the water. When the disk is pulled away it is found that the *under surface is wet*, thus showing that the attraction between the solid and the liquid is greater than that between the particles of the liquid. This is true in all cases where *a liquid wets a solid.*

If mercury be used instead of water, it is found that *a greater force* will be needed to pull the plate away from the mercury and also that the *under side of the plate is now dry.* In the latter case we find that the attraction between the solid and the liquid is less than the attraction between the particles of the liquid, as is always the case *where a liquid does not wet a solid.*

85. Capillary Phenomena. If a solid be immersed in a liquid which wets it, the liquid rises about the sides of the solid, and the surface of the liquid is concave upward. If the solid be in the form of a tube, the liquid rises into the tube to a certain height, which varies inversely as the diameter of the tube, and forms as its upper surface in the tube a meniscus of liquid with its *concave side upward.* If the liquid do not wet the solid, it is depressed about the solid, or in case of a tube it is depressed to a certain depth, varying inversely as the diameter of the tube, below the level of the liquid in the vessel, and the surface of the meniscus is *convex upward.* Since phenomena of this class are most clearly shown in the case of fine, hairlike tubes, they are called *capillary phenomena*, from *capillus*, a hair. Fig. 48 shows the action when clean glass tubes are immersed in water and mercury respectively.

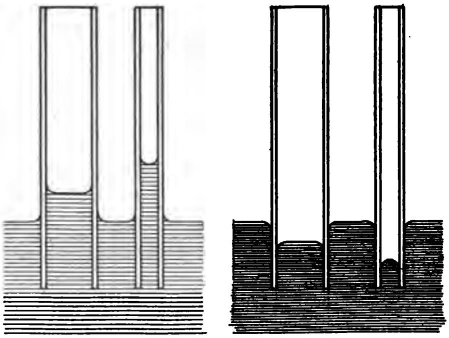

FIG. 48.

The principal facts concerning capillary phenomena are briefly these:

(*a*) In tubes of less than 2 mm in diameter the elevation or depression varies inversely as the diameter of the tube.

(*b*) The elevation or depression is independent of the pressure to which the liquid is subjected, and also independent of the thickness of the tube.

(*c*) An *increase* in the temperature of the liquid *causes a decrease* in the elevation or depression of the capillary column.

(*d*) The elevation or depression depends upon the nature of the liquid and tube in contact. Clean water on clean glass gives an elevation greater than that of any other liquid, while pure water in a steel tube or against a silver plate gives neither elevation nor depression.

Examples of capillary action are seen in the action of blotting paper absorbing ink, in the lamp wick supplying the flame with oil, and in the swelling of a tub or barrel if filled with water when about to fall to staves. A cotton or hemp rope, if wetted, absorbs water, increases in diameter and diminishes in length; at the same time the temperature of the rope rises from 2° to 10° C. Workmen drive wedges of dry wood tightly into holes or slits cut in large stones and then wet the ends of the wedges. The increase in the size of the wedge is sufficient to burst the stone.

Besides the capillarity of liquids there seems to be an analogous phenomenon in the case of metals. Joseph Henry discovered that mercury would siphon through a bar of lead as water through a towel, and silver has been shown to pass into the pores of copper when the two metals are heated.

86. Molecular Range. In accordance with the assumption that molecular forces are exerted over insensible or molecular distances, it follows that each individual molecule becomes a center from which it exerts its molecular attractions and repulsions over the number of molecules included in its sphere of influence. Let the radius of this sphere be ϵ; then ϵ denotes the limit beyond which the molecule neither influences nor is influenced by its neighbors. Within this sphere, however, it is attracted equally on all sides and hence remains in equilibrium.

In order to determine the value of this quantity ϵ, Quincke employed a glass plate (Fig. 49), one half of which was coated with a wedge-shaped layer, *AC*, of pure silver. On inserting the plate in water, with the silver film vertical (Fig. 50), the water rose against the glass above the level of the water in the dish to a definite height indicated by *D*. At the beginning of the silver film *C*, the elevation gradually fell away with increasing thickness of the silver, until at a point *B*, where the thickness reached a value of 0.000005 cm, the angle of contact became 90° and the capillary effect disappeared entirely. At this point the molecules of the glass ceased to influence the molecules of water through the silver, hence the value of ϵ, the molecular range, is commonly given as 0.000005 cm.

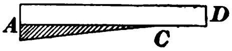

FIG. 49.

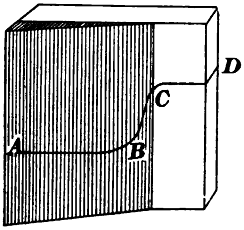

FIG. 50.

Recent investigations by Chamberlain show that this value is much too large, and that the true value of ϵ is 0.00000015 cm.

87. Surface Tension. It has been shown that within the limiting distance ϵ the molecules attract their neighbors and are attracted by them, and that a molecule situated in the body of a liquid will be in equilibrium by virtue of the equal attractions on all sides. Consider now a molecule nearer the surface of a liquid than the molecular range ϵ. In this case the horizontal attractions will be equal in all directions, but the vertical attractions are unequal; the resultant being an unbalanced component toward the interior of the liquid. At the surface of the liquid this resultant force, normal to the surface, reaches a maximum, and the mass of the liquid (Fig. 51) behaves as if surrounded by an elastic bag under stress, tending to contract indefinitely and compress the liquid into as small a volume as possible. This is due to what is known as *surface tension*.

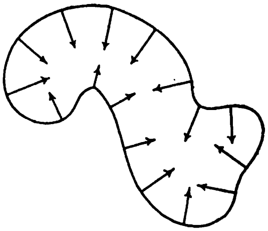

FIG. 51.

The surface of a liquid is therefore a seat of potential energy, since in order to force a molecule from the interior of the liquid out into the surface film, work must be done in moving the molecule through the distance ϵ against the forces tending to draw it back into the interior. An increase in the area of the surface film, therefore, means an increase in its potential energy, and since potential energy always tends to become a minimum, it follows that if a liquid mass be freed from the action of other forces, it will assume a form such that its surface will be a minimum, and its volume a maximum; that is, it will assume the form of a sphere. This condition is readily realized by placing drops of olive oil in a mixture of alcohol and water of the same density as the oil. The drops are thus freed from the action of gravity and float as spherical globules in a medium of their own density. If by any device the globule be prevented from assuming a spherical form, it will still take a form presenting the minimum area consistent with the conditions imposed upon it.

Surface tension is exemplified in the shape of the dewdrop, in the forms of falling drops of liquid, and in the manufacture of shot, where molten lead, poured through a fine sieve at the top of a high tower, is broken up into small globules which harden as they fall through the air, and are caught in a tank of water beneath.

Again, small heavy bodies that are not wetted by a liquid may be placed upon the surface of the liquid and float upon the surface film. Thus needles may be made to float upon water, *so long as the film is not broken*, in which case they sink at once. The same principle is illustrated in the case of small insects which run upon the surface of the water, their slight weight being insufficient to break through the surface film.

88. Experiments on Surface Tension. If two small pieces of wood be floated near each other upon the surface of clean water and a drop of oil be touched to the water between them, they fly apart to the sides of the vessel, as though drawn by a spring. The addition of the oil reduces the surface tension of the liquid film at that point, and the water film tears apart.

If a plate of clean glass be wetted with clean water, the water will spread out into a thin layer over the entire plate. If the plate be not absolutely clean, the water will gather up into irregular masses with rounded edges. If now a drop of alcohol be touched to the water layer, the water film will be seen to break at the point of contact, and gradually draw away from the alcohol drop, leaving a dry space on the plate around that point.

If a ring of stiff wire (Fig. 52) be dipped into a soap solution and withdrawn, a film of the solution will adhere to it for several minutes. In this film, which is really *two films* placed back to back, may be seen the motion of the liquid particles seeking new positions as the tension in the film changes. If we drop a loop of silk thread upon this film, it floats about freely upon it; if the film inside the loop be broken by touching it with a hot wire, the loop suddenly flies out into a circular form, showing that the tension in the film is equal in all directions. This circular loop still floats freely in the film, and behaves like an elastic hoop of steel.

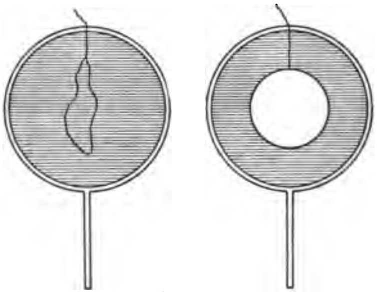

FIG. 52.

If small pieces of clean gum camphor be throw upon the surface of clean water, the little particles begin a most lively and erratic motion. Each little piece spins with great vigor and at the same time sweeps over the surface, the larger ones gathering in the smaller ones. The gum camphor dissolves slowly in cold water, and the surface tension of the water film is thereby weakened. The spin is due to unequal dissolving on the surface of the camphor particle. If the water be warmed, it spins faster; if the surface be touched with a trace of oil, the motion ceases instantly.

If several small, clean, wood or paraffin balls be thrown upon clean water, they seem to attract each other and collect into a little cluster. If a number of similar balls be smoked with lampblack and then placed in the same dish, they also attract each other, but the clean balls and the smoked balls seem to avoid each other.

A small cylinder of fine wire gauze, if immersed in water and lifted out in a horizontal position, retains the water in it and may be carried about the room. On breaking the water film at one point in the gauze by blowing upon it, the water begins to run out. The flow may be checked by shaking the water so as to restore the film, thus preventing the entrance of the air.

89. Measurement of Surface Tension. If a liquid exist in the form of a free film, then the two sides of the film exhibit surface tension in like degree and the film tends to contract indefinitely unless prevented by the application of an external force. If such a film be formed by dipping a rectangular frame of fine wire into a liquid and carefully raising it so as to keep its two sides vertical, the film tends to contract and draw the frame back into the liquid unless this contractile force be balanced by an external force. This affords a means of measuring surface tension.

If now frames of different width be taken and the forces needed to keep the frames in equilibrium be determined in each case, it will be found that the force is always *proportional to the width of the double film.* Let l represent the width of the frame, then the contractile force of the film is

$$F = T \times 2l \tag{182}$$

where T is a constant for any given liquid and is called the *surface tension* or the *capillary constant* of the liquid.[1] It is to be noted that in the case of surface tension, the term *tension* is used in the sense of *force per unit length* rather than *force per unit area* as usual. The unit of surface tension is one dyne per centimeter.

The following table shows the value of the surface tension in dynes per unit width of film, for the various substances mentioned. The values are mostly those given by Quincke.

1) For experi[illegible] determination [illegible]
Surface tension [illegible]cise 26, 27.

TABLE V

Surface Tensions of Various Substances

Mercury against air	535.0
Water against air	81.0
Olive oil against air	36.9
Alcohol against air	25.5
Mercury against water	418.0
Olive oil against water	20.6
Turpentine against water	11.6

90. Capillary Action as Related to Surface Tension. Let a tube of radius r (Fig. 53) be inserted in a liquid density d. Let the mean elevation of the liquid be h, and the angle of contact with the tube be α. Then the vertical component of the force due to surface tension T must be balanced by the weight of the liquid column of height h.

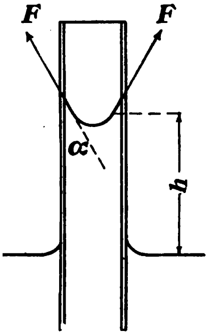

Fig. 53.

The width of the film around the tube is $2\pi r$ cm, and the total force in the direction indicated is $2\pi rT$; the vertical component is $2\pi rT\cos\alpha$.

The weight of the liquid supported is πr^2hdg; hence for equilibrium we have

$$\pi r^2hdg = 2\pi rT\cos\alpha \tag{183}$$

whence

$$h = \frac{2T}{rdg}\cos\alpha \tag{184}$$

For clean water on clean glass, the angle of contact is approximately zero, and

$$h = \frac{2T}{rdg} \tag{185}$$

For mercury, α is about 132°; h is negative, and the surface is depressed.

When we consider that the surface tension T, *decreases* with *increase* of temperature, it is seen that the above formula accounts for the inverse relation between capillary action and temperature.

For the elevation between two parallel plates distant u from each other the computation is similar to that above. The elevation or depression is

$$h = \frac{2\,T \cos \alpha}{udg} \tag{186}$$

or one half as great as for a tube of diameter u.

If two plates be joined at one edge and inserted in the water, the liquid rises high along the line of contact and falls off as the plates separate. The upper line of the fluid takes the form of an hyperbola, as shown in Fig. 54.

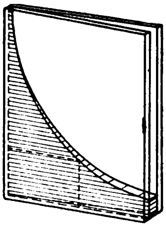

FIG. 54.

91. Angles of Contact. From the table in Art. 89, it is seen that there exists at the surface of separation between a liquid and a gas, or between two liquids, a surface stress or surface tension which is a constant for the same substances. Thus there exists in the surface film of olive oil in contact with air, a surface tension of 36.9 dynes per centimeter width of the film, while for water and air the surface tension is 81 dynes per centimeter width. If now a liquid be brought into contact with a second liquid in the presence of air, then for equilibrium, the three surface tensions should form a triangle of forces, and theoretically the angles between the forces should be constant. If, however, one of the forces should chance to be greater than the sum of the other two, then clearly no triangle is possible, and the system cannot come to equilibrium.

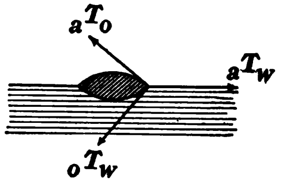

FIG. 55.

Thus in Fig. 55 is shown a drop of oil placed upon the surface of clean water. Then at any point upon the length l of the horizontal edge of the drop there are acting the three forces, $_aT_wl$ between the air and the water, $_aT_ol$ between the air and oil, and ${}_oT_wl$ between the oil and water, directed in each case as indicated by the arrows. But from the table

$${}_aT_w = 81 \text{ dynes per centimeter width}$$
$${}_aT_o = 36.9 \text{ dynes per centimeter width}$$
$${}_oT_w = 20.6 \text{ dynes per centimeter width}$$

hence

$${}_aT_w > {}_aT_o + {}_oT_w \tag{187}$$

This shows that when a drop of oil is placed upon clean water, the surface tension between air and water is so great as to overbalance the other two surface tensions combined, and the oil is dragged out in all directions, forming a film of infinitesimal thickness over the entire surface of the water.

If a liquid meet a solid in the presence of air, it will in general meet it in a definite angle which is constant for the two substances. This angle is called the *angle of contact*, and depends upon the nature of the substances in question. For pure water on clean glass the angle of contact is approximately zero. For pure water on clean steel or clean silver the angle of contact is about 90°. For clean mercury against clean glass it is about 132°.

92. Behavior of Films. If a film of soap solution be made to assume a curved form, there will always result a normal pressure directed towards the concave side. It may be shown mathematically that for a single cylindrical film, of radius R, and surface tension T, the normal pressure toward the curved side is given by the equation

$$P = \frac{T}{R} \tag{188}$$

In words this equation says that the normal pressure in a curved film is directly proportional to the surface tension, T, and inversely as the radius of the film, R. In general the curvature of any surface at any point may be expressed in terms of two radii of curvature, the planes of curvature being at right angles to each other. If R_1 and R_2 be these radii, then the normal pressure of any curved film is the sum of that due to each curvature separately, or

$$P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{189}$$

In a soap bubble the two radii are equal, and there are also two films back to back, hence the normal pressure exerted upon the air enclosed in a bubble is

$$P = 4\frac{T}{R} \tag{190}$$

If the film is free to the air on both sides, the normal pressure must be zero. In a curved film this is possible only if

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0 \tag{191}$$

This means that

$$R_1 = -R_2 \tag{192}$$

or that the radii are equal and on opposite sides of the film; that is, the film is saddle-shaped.

Again, since this normal pressure is directed toward the concave side and varies inversely as the radius, we are able to understand the motion of drops of water and of mercury in conical tubes. The water will move toward the smaller end of the tube; that is, in the direction of the greater normal pressure. For the same reason, the mercury globule will move toward the larger end of the tube.

If stout wire frames, representing the outlines of geometrical figures be dipped into soap solution, a large number of curious and beautiful film figures result. Whenever three such film surfaces meet along a line, the included angles will all be 120°, since the three forces are all equal.

CHAPTER X

SOLUTION AND DIFFUSION

93. Solution. Closely allied to the phenomena of capillarity and surface tension are the phenomena of solution. Ostwald defines solutions as "homogeneous mixtures which cannot be separated into their constituent parts by mechanical means." Gases have unlimited power of solution. One gas dissolves in another in all proportions so long as they do not unite chemically, and the homogeneous mixture manifests the sum of the properties of the two constituents.

Liquids dissolve gases without exception, although the readiness with which such solution occurs, varies greatly with the nature and condition of the substances. In a true solution of a gas in a liquid, the gas may be entirely removed from the liquid by diminishing the pressure or by raising the temperature, and in such solutions the quantity of gas dissolved by a given mass of liquid is proportional to the pressure to which the gas is subjected. Examples of this sort of solution are found in solutions of carbon dioxide, of air, or of ammonia gas in water. In other cases, as in the solution of hydrochloric acid gas in water, the dissolved gas is not entirely removed from the liquid, and it is assumed that a chemical change has resulted.

The solution of one liquid in another occurs in many cases, but is dependent upon the nature of the substances. Here also there are two distinct classes of solution. Thus some liquids, as alcohol and water, dissolve in all proportions, forming a homogeneous mixture. Ether and water, on the other hand, dissolve in each other, *but in limited proportions.* Thus water will dissolve about ten per cent of ether, but if more ether be added, the excess remains undissolved. Ether will dissolve about three per cent of water, but beyond this the water remains separate from the ether.

A third division contains those liquids that do not dissolve in each other at all. This is a relatively small number, since even those liquids that seem insoluble yet leave traces in the solvent. Thus the fact that water when shaken with the volatile oils retains the characteristic odor of those oils seems to show that even here solution has occurred in slight degree.

Again, mixtures of insoluble liquids, as water and the fixed oils, may be made by the addition of some such substance as gum acacia or gum tragacanth, in which the oils are broken up into exceedingly small globules that float in the water. Such mixtures are called *emulsions*. Milk is a natural emulsion. Some emulsions separate on standing or when subjected to mechanical action, as seen in the separation of cream from milk.

94. Solution of Solids. Many solids when immersed in a liquid gradually disappear and form a new homogeneous liquid. The solid is said to dissolve in the liquid, and the new liquid is called the *solution*. A liquid that dissolves a solid is called the *solvent*. Many salts are soluble in water. The quantity of a substance in solution may vary from zero up to a certain limit, beyond which the solution has no further action upon the substance in question. Such a solution is said to be *saturated*. The amount of a solid that may be dissolved in any solvent varies with the temperature. If the temperature of a saturated solution in contact with its salt be changed, either some of the dissolved solid separates out or more of the undissolved solid goes into solution. Generally a solvent will dissolve more of a solid when hot than when cold although there are exceptions to the rule. Thus the solubility of sodium sulphate *increases* with the temperature up to 33° C, but beyond that temperature its solubility *decreases*.

If a solution, either by evaporation or reduction of temperature, be made to contain more than its normal quantity of a solid, it is said to be *supersaturated*. If a particle of the undissolved solid be dropped into the supersaturated solution, the excess of solid in the solution crystallizes out at once (Art. 195).

95. Free Diffusion of Gases. Dalton's Law. Let a tall glass jar be inverted and filled by upward displacement with illuminating gas, and placed upon a similar jar filled with air, with the mouths of the jars together. We shall thus have the two jars filled with separate gases, and since the lighter gas is on top, no mingling of the gases can be produced by the action of gravity. If now the jars be left in position for a quarter of an hour, we shall find, on testing the contents with a lighted splinter, that there is an explosive mixture of illuminating gas and air in each jar. This illustrates diffusion of gases, and this result can be explained only on the hypothesis that the molcules of the gases are in motion and that they have therefore wandered through the entire space, the heavier gas rising into the upper jar and the lighter gas descending into the lower jar.

After the gases are uniformly diffused it will be found that the pressure exerted by the mixture is the sum of the pressures exerted by its constituent parts. Thus if we allow 10 volumes of illuminating gas and 15 volumes of air each at atmospheric pressure p to diffuse uniformly, without change of temperature, through a space of 25 volumes, then, according to Boyle's law, the air would exert a pressures of $15p/25$, and the gas $10p/25$, and their combined pressure, would equal that of the atmosphere outside. This important relation was first established by Dalton and is known as *Dalton's law.* It may be stated as follows: *A mixture of two or more gases having no chemical action upon each other, exerts a pressure equal to the sum of the pressures which would be exerted by each of the constituent gases separately if allowed to fill the containing vessel alone at the given temperature.*

It thus appears that each gas behaves as if no other gas were present, the only effect being a slightly diminished rate of diffusion, owing to the mutual molecular collisions. In general, the properties of such a mixture are found to be the sum of the properties of the various gases composing the mixture.

96. Diffusion of Gases through Porous Partitions. Atmolysis. A tube, Fig. 56, is partly filled with water, and the right arm closed with a porous cup. Over the porous cup is lowered an inverted beaker filled with hydrogen or illuminating gas.

The water sinks in the right arm and rises in the left, showing an increase of pressure in the cup. After the beaker has remained in position for a minute or two, suddenly remove it. The water now *rises* in the right-hand tube and is *depressed* in the left. The explanation of these two experiments is very simple. In the first, the lighter illuminating gas diffuses *inward* more rapidly than the air diffuses *outward*, and an increase of pressure in the cup results. In the second case the gas now inside the cup, being lighter than the air outside, diffuses *outward* more rapidly than the air diffuses *inward*, causing a reduction of pressure in the cup.

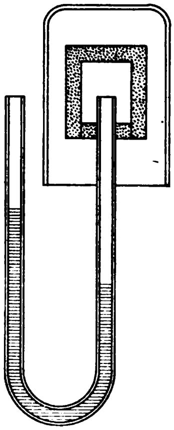
FIG. 56.

The differences in rates of diffusion for different gases have been utilized for separating a gaseous mixture into its constituent parts. Thus if we pass a mixture of hydrogen, nitrogen and oxygen through a porous tube made from the stems of clay tobacco pipes, and maintain a vacuum about the outside of the tube, we shall find that the hydrogen, being the lightest, will diffuse most rapidly through the walls of the tube, leaving the nitrogen and oxygen behind. Of course some nitrogen and some oxygen escape also, but the mixture transmitted by the tube is relatively richer in the heavier constituents, as indicated by equation (180). Rayleigh and Ramsey were able by this means to separate argon from atmospheric nitrogen. This process of separation of gases was first used by Graham, and was called by him *atmolysis*.

97. Diffusion of Gases through India Rubber, and through Red-hot Metals. In 1831 Mitchell observed that toy balloons made of india rubber collapsed much sooner when inflated with carbonic acid than when filled with air or even with hydrogen. Graham, who studied the phenomenon, found that while one volume of nitrogen would pass through a sheet of india rubber in a given time, 2.56 volumes of oxygen, 5.5 volumes of hydrogen and 13.58 volumes of carbonic acid would pass through in the same time. Also the speed with which gases pass through rubber increases very rapidly with increase of temperature.

These remarkable facts do not seem to be connected in any intimate way with the transmission of gases through porous partitions. No simple relation connects the densities of the gases with their speeds of diffusion, as in the case of porous septa, and the process of transmission appears to be an entirely different one. The most probable explanation seems to be that the rubber is capable of absorbing and retaining certain amounts of the various gases with which it comes in contact, the amounts increasing rapidly as the pressure increases. After the surface layer of the rubber has become saturated, as it were, with the gas in question, this condition is then passed on from layer to layer of the rubber, until the outside layer is reached. Here, since the pressure is less, the rubber is unable to retain all the absorbed gas, and some of it escapes into the adjacent space.

Red-hot metals transmit gases with great facility. The poisonous carbon monoxide passes freely through cast iron at red heat, and from the red-hot cast iron coal stove this deadly gas leaks into the room, almost as water from a sieve. Hydrogen diffuses through a red-hot platinum tube, whose walls are 1.1 mm thick, at the rate of *490 cc per minute for every square centimeter of surface.* Silver at high temperatures transmits oxygen readily. A glowing tube of palladium, through which is carried a mixture of CO and H, separates these gases completely, the hydrogen being transmitted and the carbon monoxide being retained. It thus appears that glowing platinum and palladium act as "semi-permeable membranes" for certain gases, just as certain solid substances do for liquids, in that they allow some substances to pass freely and refuse transmission to others. This peculiarity is of great importance in osmotic phenomena, as we shall see later. In all cases of such transmission through rubber or glowing metals, we seem to have to do with a species of solution of the gas on the one side of the partition, and of evaporation of the gas from the other side. The same process seems to account for similar behavior in the case of gases and liquid films.

98. Free Diffusion of Liquids. If two liquids that do not react chemically upon each other be left in contact with each other, they will of themselves begin to mingle at once, and continue until they form one homogeneous liquid throughout. Thus, if a solution of copper sulphate be placed in the bottom of a small jar, and carefully covered with distilled water, so the line of separation is well defined, and the jar be left undisturbed for a few days, we shall see that the blue color of the copper sulphate has risen into the clear water above, and that the line of demarcation is no longer sharp between the liquids. The color of the copper salt at the bottom has also become slightly less dense than at first, and the two liquids seem tending toward a uniform color.

This process is called *diffusion*, and while resembling the related phenomenon in gases, its progress in liquids is exceedingly slow. For example, if the jar containing the copper sulphate in the above example be made a meter high, the lower half filled with the solution and upper half containing pure water, it would take more than ten years for the solution to assume a uniform color throughout. If the jar were one centimeter high, it would require about ten hours, the time for uniform diffusion varying as the square of the length of the liquid column.

The speed at which a given solution will diffuse through the pure solvent depends upon the nature of the salt and of the solvent, upon the temperature, and to a slight degree upon the strength of the solution. From extended experiments it has been found that those salts having the highest electrical conductivity have also the highest velocity of diffusion.

99. Diffusion through Membranes. Osmosis, Crystalloids and Colloids. As we have seen, if two solutions of different strength be brought into contact, a condition of equilibrium cannot, in general, be maintained. A movement of the dissolved substance sets in *from the concentrated to the dilute* solution, and continues until it is uniformly distributed throughout the liquid. If, however, we enclose the solution in a vessel fitted with a manometer tube, and provided with a bottom of some porous substance, as parchment or animal membrane, and immerse the whole in pure water, we shall find that equilibrium is possible.

The porous membrane refuses transmission to the molecules of the dissolved substance, while allowing the solvent to enter freely. As a result, the solvent crowds in through the membrane and creates an internal pressure, as shown by the rise of liquid in the manometer. This crowding in of the solvent continues until the pressure reaches a definite value, depending upon the strength of the solution. After this, the tendency of the molecules of the solvent to enter the cell seems to be balanced by the internal pressure, and equilibrium ensues.

This unequal diffusion through porous septa is called *osmosis*, and the membrane is termed a *semi-permeable membrane.* The limiting pressure beyond which no more of the solvent enters the cell, is called the *osmotic pressure* for the subtance, *at that temperature* and *for that concentration.*

The phenomenon may be illustrated by the following experiment. A conical vessel attached to a long tube is closed at its larger end by a piece of bladder or parchment firmly tied on. When the vessel is filled with sugar solution to the lower end of the tube, and immersed in a vessel containing water as shown in Fig. 57, the liquid in the tube rises to a considerable height above the level of the water in the outer vessel.

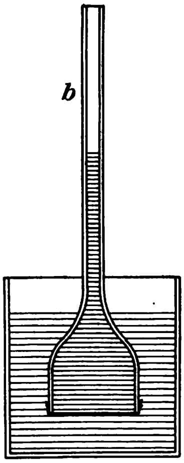

FIG. 57.

Through the experiments of Pfeffer it was discovered that the best results are to be obtained by attaching the tube to a closed clay cell, the pores of which are filled with a precipitate of copper ferrocyanide. The precipitate is pervious to water but impervious to the dissolved substance. With such a cell, filled with a 3.3 per cent solution of potassium nitrate, Pfeffer obtained an osmotic pressure of 436.8 cm of mercury, *or more than 5.7 atmospheres.*

Those substances which pass through animal membranes most readily, such as mineral acids and neutral salts, are generally known in the crystalline form and hence have been called *crystalloids*. Substances like gums, tannin, albumen, starch, etc., which are amorphous in character and do not pass through so readily are called *colloids*. The effects of crystalloids when dissolved in water are very marked in the changes produced in the properties of the solvent. Thus any crystalloid dissolved in water *diminishes its vapor pressure*, *lowers its freezing point and raises its boiling point*. Colloids, on the other hand, when dissolved in water produce scarcely any such effects. Colloidal solutions in general represent loose mechanical mixtures from which in many cases the substance held in solution may be precipitated by a slight trace of acid or alkali. When mixed with small quantities of water the colloids form jellies, in some of which the structure is so coarse as to be visible under the microscope. This is notably so of the colloidal solutions of the salts of gold, in which the suspended particles of gold form the objects for ultra-microscopic vision by means of transverse illumination. While many of these colloidal jellies transmit crystalloids almost as readily as pure water, they offer great resistance to the diffusion of other colloids.

100. Osmotic Pressure. We have seen that the entrance of water through the semi-permeable membrane into the osmotic cell may be prevented by subjecting the enclosed solution to sufficient pressure. This pressure is called the *osmotic pressure*, for the substance in question under the conditions of the experiment, and is of great importance since the properties of the solution, such as its vapor pressure, its boiling and freezing points, are immediately calculable as soon as this pressure has been determined. Osmotic pressure is also intimately related to the transmission of fluids in the living cell of plant or animal tissues. Thus it has been shown that such a cell when placed in concentrated salt solutions, has its liquid contents diminished by the removal of water; if placed in a solution whose osmotic pressure is less than that of the cell, the cell and its contents are distended by the addition of water.

Oil globules of extreme smallness floating in water tend more readily to pass bodily through the pores of animal membranes, if some alkali be mixed with the water, since they have a soapy covering and are thus passed through the pores like so much water. (Daniell.)

The following conclusions concerning osmotic pressure are stated by Ostwald as reasonably well determined for dilute solutions:

(*a*) The osmotic pressure depends upon the nature of the substance.

(*b*) The osmotic pressure is proportional to the concentration of the solution, or inversely proportional to the volume in which a definite mass of the dissolved substance is contained.

(*c*) The pressure for a given concentration is proportional to the absolute temperature.

(*d*) Quantities of dissolved substances not electrolytes which are in the ratio of their molecular weights exert equal pressures at equal temperatures.

(*e*) The pressure is independent of the nature of the membrane provided the membrane be impervious to the dissolved substance.

101. Dialysis. The division of substances into *crystalloids* and *colloids* has already been mentioned. A characteristic property of colloids is, that while they pass through porous septa with difficulty *themselves*, they offer no marked resistance to the diffusion of *crystalloids*, but are more or less impervious to other colloids. A result of this property is, that if a mixture of crystalloid and colloid substances be separated from pure water, by a colloidal membrane of a different kind, the crystalloids will soon diffuse into the water, while the colloids will remain behind.

This principle has been applied to the separation of crystalloidal poisons from a heterogeneous mass of organic matter in which their presence is suspected. The apparatus as employed by Graham consists of a hoop, over one side of which has been stretched a piece of bladder or parchment paper, put on wet and held in place by a string. This is slipped inside a second hoop, thus forming a shallow dish with a colloidal bottom. This is now floated in a vessel containing distilled water and the substances to be examined are placed in a thin layer on the membranous bottom. If crystalloids be present, they will diffuse through into the distilled water, while the colloids are left behind. The water in the lower vessel may then be examined for the suspected poisons by ordinary analysis. The floating dish is called a *dialyzer*, and the process is termed *dialysis*.

Problems

1. How high will water stand above hydrostatic level in a tube 0.057 cm in diameter, assuming the angle of contact to be very small?

2. How much will the level of mercury be depressed in a glass tube 0.067 cm in diameter, the surface tension being taken as 540 dynes per centimeter and the angle of contact as 135°?

3. What is the pressure within a soap bubble 12.5 cm in diameter if the surface tension of the liquid be taken as 80 dynes per centimeter, and what is the total force exerted by the film on the gas within?

SOUND

ORIGIN AND PROPAGATION

CHAPTER XI

NATURE OF SOUND

102. Definitions. Under sound are studied those manifestations of energy which primarily appeal to the ear. This branch of physics has to do with the study of vibratory motion in ponderable, elastic media. The phenomena of sound differ from those of heat, light, and electricity in this respect, that the cause of the phenomena is definitely known, while in heat, light, and electricity, the cause is assumed.

In common language the word *sound* is used in two distinct senses. It may mean the sensations reported to the brain by the auditory nerves, or it may refer to the external cause of those sensations. The psychologist and physiologist are concerned with the phenomena of sense perception. The physicist is interested in the external disturbance *producing the sense perception;* the conditions of its origin, its mode of propagation, and the variations in the nature of the disturbance corresponding to certain differences in the sensation produced. Accordingly sound is defined by the physicist as *that form of vibratory motion which may be perceived by the ear.* According to this definition sound may exist entirely independent of an ear to perceive or a brain to comprehend.

103. Origin of Sound. Sound originates in a vibrating body. "Sound and movement," says Blaserna, "are so correlated that one is strong when the other is strong, one diminishes as the other diminishes, and the one stops when the other stops." A guitar string plucked aside and released gives a musical note, and at the same time seems to spread out into a broad band with a hazy outline, which diminishes to the original size of the string as the sound dies away. The tremulous motion of a bell may be perceived by placing the hand upon it while it is sounding. A long glass tube, if grasped by the middle and rubbed with a moistened cloth, gives forth a soft musical note, while the vibratory motion may be plainly felt by the hand. The air column in an ordinary tin whistle is thrown into vibratory motion when the whistle is blown. The whistle may even be blown by water and the instrument will give forth a soft clear note, produced by the vibration of the stream of water.

104. Wave Motion. We have seen (Art. 33) that a simple harmonic motion compounded with a uniform motion in a straight line produces a "sine curve." Such a curve (Fig. 17 *bis*), may be regarded as arising from a series of equidistant particles arranged along the axis M_0M_2, each executing simple harmonic motion at right angles to this axis, but having a common difference of phase; that is, each particle to the right of M_0 crosses the axis going in the positive direction, $\frac{1}{16}$ *of a period later than its left-hand neighbor.* The result of such a system of moving particles is the sinuous form, known as a ***sine curve*** or ***simple wave.***

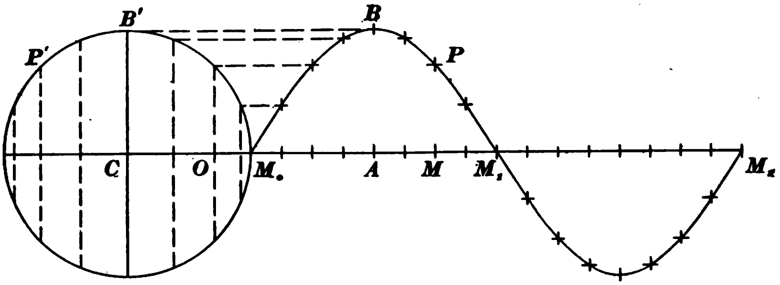

FIG. 17 *bis*

During the time T, required for any [illegible] describe a complete vibration, the disturbance wil[illegible] o the right, the distance from M_0 to M_2: [illegible] *second* *vibration*, M_2 begins its *first*. Since M_2 marks the extreme advance of the wave disturbance at this instant, it may be said to lie on the *wave front*. By this definition special attention is directed to the limiting position reached by the disturbance. However, we frequently use the term *wave front* when speaking of an extended series of moving particles in space and, regardless of the limit reached by the disturbance, we are accustomed to say that a wave front is the *continuous locus of all points in the same phase of vibration.*

The distance M_0M_2 is called a *wave length* λ, and is related to the *velocity*, V, of *the wave*, and the time of vibration, T, of the particle, by the equation

$$\lambda = VT = \frac{V}{n} \qquad . \qquad (193)$$

where n denotes the *frequency* or the *number of vibrations* made by a particle *in one second.*

In the case considered the particles are conceived to vibrate in paths at right angles to the line of propagation of the wave; such a wave is termed a *transverse wave.* Examples are seen in the waves produced in a soft rope when one end is vibrated quickly and regularly in a line at right angles to its length.

If the particles vibrate back and forth *in* the line of propagation of the wave, the wave is called a *longitudinal wave.* Such a wave is produced by suspending a long spiral coil of brass wire between two fixed supports and producing a compression in it by forcing the spirals at one point closer together. The compression runs the entire length of the coil.

105. Characteristics of Wave Motion. The fundamental characteristic in wave motion is the continuous handing on, from point to point, in an elastic medium, of a periodic disturbance maintained at the source. Such a disturbance produces a series of waves which follow each other at definite intervals and which constitute what is known as a wave train. It is to be observed that wave motion *transfers energy* from one point to another, by means of the *motion of the particles of the medium.*

The individual particles oscillate about their positions of rest, while the *wave form* runs forward. The distinction between the motion of the particle and the motion of the wave is of fundamental importance. It may perhaps be best illustrated by the motion of a wave passing over a field of grain. The wave form runs forward while the individual head of grain simply swings back and forth in a plane parallel with the line of propagation.

Other characteristics of wave motion may be noted as follows:

The disturbance requires time to travel from one point to another.

A medium is required for transmission of the disturbance.

Waves are *reflected* on meeting an obstacle, the angles of incidence and reflection being equal.

The direction of the wave is changed, that is, the wave is *refracted*, on entering a medium in which the speed of propagation is different.

Two systems of waves may be added together so as to reënforce each other, if crest meet crest and trough meet trough; or they may be added so as to annul one another and produce rest, when the crests of one system meet the troughs of the other. This is known as *the interference* of wave systems.

Waves, on meeting small obstacles, bend round corners and behind the obstacles, form a series of points of maximum and minimum disturbance. This phenomenon is known as *diffraction.*

Two waves may differ from each other in *amplitude*, in *wave length* or in *wave form.*

106. Characteristics of Sound. A sounding body transmits its vibrations to the surrounding medium and a system of sound waves is the result. The waves of sound are longitudinal in character, the vibrations of the particle taking place in the line of propagation. A comparison of the characteristics of wave motion with those of sound reveals the following points of similarity.

Sound requires time for its transmission from point to point; in other words, sound travels with a definite velocity, which depends upon the nature and condition of the medium. Examples of this finite velocity of sound are seen in the interval elapsing between the flash of a distant gun and the report of the discharge ; or between the appearance of the puff of steam and the sound of the whistle of a distant locomotive, or in the case of the distant woodman who has his ax raised for a second blow before the sound of the former one reaches us.

Sound requires a continuous, elastic and ponderable medium. Sound is not transmitted in a vacuum. An electric bell suspended by rubber cords from the inside of a bell jar is silenced on the exhaustion of the air from the jar, although the clapper may still be seen to be moving. On readmitting the air the sound is heard again. Unless the medium be elastic the energy originally imparted to the medium by the vibrating body is largely lost in heat, as in the case of impact between inelastic solids. The medium must also be ponderable ; that is, it must have weight. Sound is a *material* phenomenon. It is produced by the vibrations of material particles, and requires a material medium for its transmission. Liquids transmit sound better than gases, and solids better than either.

Sound is reflected. Whispering galleries are simply large rooms in which the curved surfaces of the walls or roof serve as mirrors for the concentration of the sound waves at particular points. Speaking tubes are devices for preventing the dissipation of sonorous energy, by confining it to a certain narrow space, through which it passes by repeated reflections from the sides of the tube. Echoes are examples of the reflection of sound by the walls of the building, by hills or forests.

Sound may be refracted. A large lens-shaped balloon of collodion film, when filled with carbon dioxide acts as a condensing lens, for sound waves, and brings them to a focus by making the convex wave fronts concave. Beyond the focus the fronts again become convex outward, and the intensity of the sound diminishes.

Sound waves may be made to interfere. An illustration of this fact is the peculiar throbbing or beating effect produced when two tuning forks of slightly different frequency are sounded together.

107. Sound Waves Longitudinal. If a bell be struck or a tuning fork be put in motion, the layers of air immediately surrounding the vibratory system are thrown into vibrations of the same period, amplitude and form as the vibrations of the sounding body. This means that when the prongs of the tuning fork or the sides of the bell swing *outward* the layers of air are crowded together, producing a *condensation.* Since in a gas all the particles are free to move, this condensation is promptly handed on to the next layer of air particles, and by them to the next, and so on, each layer swinging *outward* in the same manner as the sounding body, only *a little later in time.* On the *inward* swing of the sounding body the layers of air immediately surrounding the body *are rarefied* by the receding body, and the pressure of the air from without promptly fills the rarefied space, only to produce a *rarefaction* where the *adjacent layers had been.* In this way the rarefaction is handed on from point to point, as the counterpart of the *condensation* that preceded it.

On the next *outward* swing of the body a second condensation is produced followed by a second rarefaction, and the phenomenon continues while the motion lasts. Now since the condensation and rarefaction each occupy one half a vibration in their production, it follows that *they together constitute the two parts of the sound wave*, and that the distance from one condensation or rarefaction to the *next* condensation or rarefaction is a *wave length*, λ, of the sound in question. Also in the condensation the particles are crowded together, and in the rarefaction they are drawn apart while the wave runs its own length during the time of a single vibration of the particles. Since these vibrations are *in* the line of propagation of the disturbance, it follows that *sound waves are longitudinal.*

108. Fundamental Differences. Loudness of Sound. Two sounds as perceived by the ear may differ in three respects. They may differ in *intensity*, in *pitch*, or in *quality.* These three fundamental differences in sense perception owe their existence to fundamental differences in the vibratory movements producing the sensations. It has already been noted that two waves may differ from each other in *amplitude*, in *wave length*, and in *wave form*. It will be shown hereafter (Arts. 108, 123, 141) that these three fundamental differences in wave motion determine the three fundamental characteristics of sounds as perceived by the ear.

Loudness or *intensity* of sound denotes in rough measure the kinetic energy of the vibrating medium as reported by the ear. Obviously in so complex a phenomenon as sense perception no rigid measure of the absolute intensity of a sound is possible. Not only does the same sound appear of different intensity to two different observers, but to the same observer on different days and in different states of health and nerve tension. Comparisons of the relative intensity of two or more sounds of the same general character are possible, however, and show that the intensity of sound depends essentially upon three things.

(*a*) *Upon the amplitude of vibration of the sounding body.* We have seen that two waves of the same frequency may differ in the relative excursions executed by the vibrating particles. From elementary considerations it may be seen that if the excursions of the particle be increased, the average velocity of the particle must be proportionally increased, since the vibrations in each case are executed *in the same time*. The velocity of the vibrating particle is therefore proportional to the amplitude. But the kinetic energy of the particle is proportional to the *square of the velocity*, and consequently to the *square of the amplitude*.

(*b*) *Upon the area of the sounding body.* The effect of area in modifying the loudness of a sound produced by a given body, is essentially a question of transference of energy. A tuning fork when once set in motion, possesses a definite amount of kinetic energy. If the fork be held in the hand, the sound produced is faint, but continues for several seconds. If the stem of the fork be placed firmly upon the top of a table the note becomes loud and strong and soon dies away. In the latter case the top of the table is thrown into co-vibration with the fork, and the volume of air that is energized by this means is greatly increased. The store of energy is spent in the production of sound in a much shorter time.

(*c*) *Upon the distance from the sounding body.* The intensity of sound varies inversely as the square of the distance from the sounding body. This is known as "the law of inverse squares," and may be demonstrated as follows: Suppose a single sharp sound has been produced at a point O, as a center. This represents a definite amount of energy. Now at any instant this energy is resident on a spherical shell of radius r, and of area $4\pi r^2$ cm². If we call the intensity of sound I, the kinetic energy per unit area of this shell, then the total energy is $4\pi r^2 I$. An instant later this energy has been transferred to a shell of radius r_1, on which the intensity is I_1, hence the total energy on the shell is $4\pi r_1{}^2 I_1$.

Therefore
$$4\pi r^2 I = 4\pi r_1{}^2 I_1 \qquad (194)$$

whence
$$\frac{I}{I_1} = \frac{r_1{}^2}{r^2} \qquad (195)$$

or the intensity of sound varies inversely as the square of the distance from the sounding body.

From a physical standpoint the intensity of a sound is best defined as the *quantity of energy passing in unit time through unit area of a surface placed at right angles to the direction of the sound.* From this definition accurate measurements of the intensity are possible, but their realization is difficult and involves considerations beyond the limits of an elementary text.

CHAPTER XII

VELOCITY OF SOUND

109. Experimental Determinations. The problem of determining experimentally the velocity of sound in air presents peculiar difficulties. The nature of the problem demands that the measurement be made in free, open air, where the disturbing effects of winds, which vary both in magnitude and direction, and of local differences in temperature, are unavoidable and beyond the control of the observer. To these are added the errors of observation attendant upon measurements depending upon sense perception.

If the measurements are made by one observer who notes and records the time of seeing a flash, and of hearing the report of a distant gun, then each observation is affected by the errors due to the reaction periods of both sight and hearing. By this is meant the time required for any observer to see and record the flash, and to hear and record the sound. The reaction period for sight is different from that for hearing. The error for each operator is termed his "personal equation." Not only do these errors vary with different operators, but they vary with the varying conditions of health of the same operator on different days, and with the intensity of the sound. No allowance is made for the time occupied in transmitting the flash, since for small distances this time is practically zero.

A study of the various errors just mentioned has influenced the selection of methods of observation. If cannon be used as the source of sound, the method of "reciprocal firing," whereby the sound is transmitted first in one direction and then in the other between two stations, has been used to eliminate in large measure the errors due to the wind. This, however, requires two observers, and introduces errors arising from differences in "personal equation" which are difficult to determine and cannot in general be wholly eliminated by exchange of observers. In the experiments of Stone in 1871, two operators, about three miles apart, recorded the time of hearing the report of a cannon placed some 640 ft distant from the first operator. On exchanging operators the error from "personal equation" was reduced to 0.02 sec.

The mean of Stone's results, reduced to 0° C, was 1090.6 ft per second, or 332.4 m per second.

More accurate determinations of this important constant, made by Violle and Vautier, give for the velocity of sound in air at 0° C the value

$$V = 331.36 \frac{m}{sec}$$

and this value is now generally accepted as the best result yet obtained.

110. Experiments of Regnault. Between the years 1862 and 1866 Regnault carried on an exhaustive series of experiments for the determination of the velocity of sound, both in the open air and in the water and gas pipes of Paris. In his researches, Regnault made use of an automatic recording apparatus, by means of which an electric current was broken at the instant of firing the gun, and the interruption of the current was recorded upon a smoked paper carried upon the drum of a chronograph. At the receiving station the sound wave entered a wide cone, at the smaller end of which it impinged upon a thin rubber membrane, and setting it in motion broke a second electric current, and so completed the record upon the cylinder of the chronograph. By this means, it would seem that the difficulties of personal equation were entirely obviated, but it was found that the membrane itself required time to receive and record the sound wave. The motion of the air particles cannot be imparted to the membrane instantly, and so a delay is caused in making the record, which is not constant, but increases as the sound grows more faint. Regnault made experiments to determine the amount of this error, and allowed for it in his computations.

In his experiments upon the velocity of sound in tubes, Regnault arrived at the following conclusions:

(*a*) In cylindrical pipes the intensity of the sound wave decreases with the distance, and more rapidly in small tubes than in large ones.

(*b*) The velocity of sound diminishes with the intensity. Loud sounds travel faster in tubes than faint ones.

(*c*) The velocity of sound in pipes increases as the diameter of the pipe increases, tending toward a limit in very wide tubes.

(*d*) The velocity is independent of the pressure, and of the mode of production of the sounds.

Regnault gave as the result of his investigations, after all corrections had been applied, the value for a faint sound in a very wide tube at 0° C

$$V_0 = 330.6 \frac{m}{sec}$$

111. Theoretical Velocity of Sound. From purely theoretical considerations concerning the condition of the medium through which the sound wave passes, Newton, in 1686, developed a formula for the velocity of sound,

$$V = \sqrt{\frac{e}{d}} \tag{196}$$

where d is the density of the medium and e its coefficient of volume elasticity. This formula applies directly in the case of liquids and solids, while for gases it leads to erroneous results, unless there be applied certain corrections, the nature of which will appear later.

In the application of this formula to gases, it is to be shown that the coefficient of volume elasticity e is, for ordinary sounds, equal to the pressure to which the gas is subjected. Thus, let P, V, and d represent the pressure, volume and density, respectively, of a given mass of air. Let the pressure be increased by a small increment dp, and as a result of this increase, let the volume be diminished by a small amount dv, *the temperature remaining constant.* Then, by Boyle's law, we have

$$\frac{P}{P+dp}=\frac{V-dv}{V} \tag{197}$$

or

$$\frac{dp}{P+dp}=\frac{dv}{V}$$

whence

$$\frac{dp}{\frac{dv}{V}}=P+dp \tag{198}$$

but the left-hand member of this equation is by definition the coefficient of volume elasticity, e (Art. 58), hence $P+dp=e$.

For ordinary sounds the change in pressure dp, produced in the condensation or rarefaction, is negligible as compared with the atmospheric pressure, and hence may be disregarded. Under this assumption our formula becomes

$$V=\sqrt{\frac{P}{d}} \tag{199}$$

where P is the reigning barometric pressure, and d the corresponding density of the air.

This would indicate that the velocity of sound is independent of the intensity, and this assumption seems to be sustained by observation in the case of all sounds where the change of pressure dp, produced by the condensation, is negligible in comparison with the barometric pressure, P. In disturbances where dp is not negligible, the velocity is represented by the formula

$$V=\sqrt{\frac{P+dp}{d}} \tag{200}$$

and increases as the intensity increases. In loud sounds, as those produced by explosions or the discharge of cannon, the change in pressure dp may even exceed P. Captain Parry, who made a series of measurements upon the velocity of sound in the arctic regions, relates that the report of a cannon was frequently heard by a distant observer, before the command to fire. Other observers have repeatedly found that very loud sounds travel more rapidly than those of ordinary intensity.

112. Application of Newton's Formula. It has been said that Newton's formula

$$V=\sqrt{\frac{P}{d}}$$

does not apply directly to the case of gases. If the values of P and d be substituted in this formula, we have for standard conditions, $P = 1{,}012{,}630$ dynes per square centimeter, and $d = 0.001293$ g per cubic centimeter. Whence

$$V=\sqrt{\frac{1{,}012{,}630}{0.001293}} = 27{,}927\frac{cm}{sec}$$

This is only 84 per cent of the velocity of sound as determined by experiment. The discrepancy between the theoretical and observed value was recognized by Newton, who sought to account for the difference by means of a number of ingenious suppositions, none of which, however, were justified.

113. Laplace's Correction. It was not until 1816 that the error in Newton's formula was pointed out and corrected by Laplace. It was pointed out (Art. 111) that for small disturbances the coefficient of volume elasticity e, in the formula

$$V=\sqrt{\frac{e}{d}}$$

could be replaced by the pressure P, to which the gas was subjected.

This deduction was made upon the assumption that Boyle's law would hold for the phenomena. This would mean that the air in which the condensations and rarefactions of the sound wave were produced should remain throughout *at a constant temperature;* or, in other words, that the condensations and rarefactions were to occur under *isothermal conditions.*

Laplace pointed out that in the case of a sound wave, where account

the compressions and rarefactions follow each other at the rate of several hundred per second, there could be no mention of isothermal conditions, since there could be no time for the air to assume a uniform temperature. The coefficient of elasticity of the air, that really does enter the equation for the velocity of sound, is therefore to be determined, under the condition that *no heat is to escape from the gas when it is compressed, and none is to come to it when it is rarefied.* Such conditions for a gas are called *adiabatic conditions*, and the coefficient of elasticity of a gas obtained under such conditions is called *the coefficient of adiabatic elasticity* (Art. 184).

Now it is well known that if a mass of gas be compressed suddenly, its temperature is raised, and the elastic tension of the gas is greater *because of this increase in temperature.* Careful experiment has shown that for air, the coefficient of *adiabatic elasticity* is 1.41 times the coefficient of *isothermal elasticity.* Hence the pressure P must be multiplied by 1.41 to represent the facts. The corrected formula is therefore

$$V=\sqrt{\frac{1.41\,P}{d}} \qquad (201)$$

When the correcting factor is introduced into the computations in the previous article we have

$$V=\sqrt{\frac{1.41 \times 1{,}012{,}630}{0.001293}}=332.3\,\frac{m}{sec}$$

which agrees admirably with the value found by experiment.

114. Correction for Temperature. Since the air is free to expand, an increase in temperature of the air as a whole will affect the *density* of the air, but will leave the pressure unchanged. If a be the coefficient of expansion for gases, then any bulk of air at 0° C will have a bulk $(1+at)$ times as large at t° C. Since the densities are inversely as the volumes, then d_t, the density of the air at t° C, will be given by the equation

$$d_t=\frac{d}{1+at} \qquad (202)$$

Hence the velocity of sound at $t°$ C will be

$$V_t = \sqrt{\frac{1.41 \times P}{d_t}}$$

or

$$V_t = \sqrt{\frac{1.41 \times P(1+\alpha t)}{d}} \qquad (203)$$

If V_0 be the velocity of sound at 0° C, then

$$V_t = V_0\sqrt{1+\alpha t} \qquad (204)$$

If we set $t = 1°$ C and $\alpha = 0.003665$, then the *increase* in velocity for a rise in temperature of 1° C is

$$\begin{aligned} V_1 - V_0 &= 331.36\sqrt{1+0.003665} - 331.36 \\ &= 0.00183 \times 331.36 \\ &= 0.606\,\frac{m}{sec} \text{ or } 23.8\,\frac{in}{sec} \end{aligned}$$

Hence the velocity of sound increases about 60.6 cm per second, or 23.8 in per second for an increase of 1° C.

115. Velocity of Sound in Solids and Liquids. For solids and liquids Newton's formula is applicable at once. For copper the coefficient of elasticity e is about 12×10^{11} dynes per squar ecentimeter and $d = 8.8$ g per cubic centimeter, therefore

$$V = \sqrt{\frac{12 \times 10^{11}}{8.8}} = 369{,}300\,\frac{cm}{sec} = 3693\,\frac{m}{sec}$$

For water the density is 1 gram per cubic centimeter and the *compression for an increase in pressure of one atmosphere is* 0.0000499; hence

$$\begin{aligned} V &= \sqrt{\frac{1{,}012{,}630}{1 \times 0.0000499}} = 142{,}500\,\frac{cm}{sec} \\ &= 1425\,\frac{m}{sec} \end{aligned}$$

The best experimental determinations of the velocity of sound in water give a mean result of 1435 m per second.

CHAPTER XIII

REFLECTION AND SUPERPOSITION OF SOUND WAVES

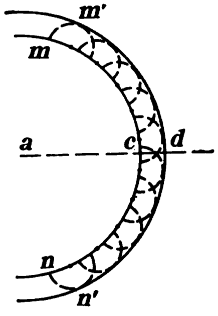

FIG. 58.

116. Huygens's Principle. Suppose a sound wave originate at *a*, as a center of disturbance. The wave will travel outward in space in all directions in the form of a spherical shell. Let Fig. 58 represent the trace on the plane of the paper of a section through this wave shell. At the end of a certain time, the wave will have reached the position *mcn.* This surface, marking the location of all points in the same phase of vibration, is by definition a *wave front.* At an instant later it will have taken the position *m'dn'*, and all points in this new wave front will be vibrating with the same motion possessed *now* by the points on the wave front *mcn.* The disturbance at the center has thus reached any point in *mcn* by the disturbance of all points in the medium through which it has passed. Hence the subsequent disturbance at any point outside the wave *mcn* is to be regarded as the resultant effect produced at that point, of all the wave disturbances originating in the individual points of the wave front *mcn* as centers.

According to *Huygens* we are to consider *every point on a wave front as a new center of disturbance, from which waves are propagated outward as from the original center, and the effect at any point external to the wave is to be regarded as the resultant of the combined action of all these elementary waves.*

If we take all the points on the wave front *mcn* as centers and about them describe small circles representing the limits reached by the elementary waves in a very short space of time, then it will be seen that these little waves unite to form a *new* *wave front*, which is a common tangent or *envelope* to them all. This is to be regarded as the new wave front. Professor Stokes has shown that the elementary waves destroy each other except at the surface of their common envelope.

117. Reflection of Sound. Sound waves are reflected regularly from any smooth surface of different density from that of the medium in which they are traveling, and the sound seems to come from some point behind the reflecting surface. By a "smooth" surface is to be understood a surface whose inequalities are small in comparison to the wave lengths to be reflected. Thus a brick wall, a hillside or even a row of trees may present a surface that is relatively smooth to the waves of sound, while for the reflection of light waves only the most highly polished surfaces are "smooth."

Tyndall showed that sound waves are reflected not only from solids and liquids but from layers of gas of different density as well. Thus a flame from a fishtail gas burner and the heated air above it, reflect sound waves to a marked degree. In the same way ascending currents of warm air over parts of the earth unequally heated by the sun, serve to reflect and scatter sound waves, as do also ascending layers of air loaded with water vapor.

In some cases the nature of the sound is modified by the nature of the reflecting surface, as in the case of a sharp, clear sound reflected by the separate bars of a picket fence. Here the separate pickets act as independent sources of sound waves and the short, sharp sound comes back as a clear musical note of definite pitch.

Echoes are produced when the reflecting surface is at a distance of more than 16.5 m. The ear cannot separate syllables occurring more rapidly than about ten per second. Hence a sound and its echo will be heard separated if the distance to the reflecting surface is greater than half the distance which sound would travel in one tenth of a second.

Applications of the principle of reflection of sound are seen in the speaking trumpet or megaphone, the ear trumpet, the stethoscope, and in the curved surface of the external ear which acts as a reflector to concentrate the sound waves into the auditory canal.

118. Reflection at End of Cylindrical Pipe. A case of special interest is found in the reflection of a sound wave at the end of a tube or pipe in which it is traveling. If the reflections occur in a closed pipe, against the solid end of the pipe, the direction of the wave is reversed, but the character of the disturbance is not reversed, a condensation is reflected as a condensation and a rarefaction as a rarefaction. This is expressed by saying that at the closed end of a pipe the sound wave is reflected *with change of sign in the velocity of the air particles, but without change of sign in the condensation.*

At the end of an open pipe, however, the result is different. When the disturbance reaches the open end of the pipe, the last layer of air having no longer an opposing resistance in front, as it had within the tube, swings outward beyond its position of equilibrium, dragging after it the adjacent layers one after another. A new disturbance thus enters the tube at the open end, in the nature of a rarefaction, and is propagated backward, while the original condensation pursues its course in the free air. This is expressed by saying that reflection occurs at the open end of a pipe, *without change of sign in the velocity of the air particles, but with change of sign in the condensation.* At the open end of a pipe, therefore, a condensation is reflected as a rarefaction, and *vice versa.* This principle will be found of great importance in the theory of open and closed organ pipes.

119. Superposition of Sound Waves. If two stones be dropped a small distance apart into still water, each is seen to become the center of a system of circular waves, widening in all directions as they run. If the two sets of waves cross each other, they are seen to pursue their own way in each case as if there were no other wave on the surface of the water. In general, when two systems of waves traverse a medium simultaneously, while the elevation or depression of either alone relative to the disturbed surface is the same as it would have occasioned if the other were not there, yet the total displacement of the surface relative to the undisturbed level is at any point and at any instant the algebraic sum of the disturbances which each of the systems would produce separately.

If an elevation of the first system be superposed upon an equal elevation of the second system, the total height of the water above the original level will be double that of one of the two waves alone. If an elevation of one system meet an equal depression of the other system, the original level is not changed.

In a similar way, sound waves, when coexistent in the same medium, form at every instant composite waves, made up of the separate systems. This superposition of wave systems is termed *interference*.

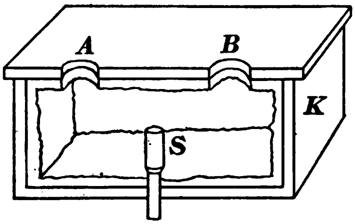

FIG. 59.

120. Principle of Interference. Suppose a small whistle to be placed in a box, heavily padded within to prevent any transmission of sound waves through its walls, and let there be in the top of the box two identically similar openings, *A* and *B* (Fig. 59), symmetrically situated with respect to the whistle. We have here two identical sources of sound, from which condensations and rarefactions proceed outward in all directions at the same instant, and such that air particles, equidistant from the two sources, on being disturbed by the individual sources *A* and *B*, have their motions outward *from* or inward *toward* their respective openings *at the same instant.* If now we consider a plane PP' (Fig. 60), normal to AB at its middle point, it is clear that two similar disturbances, starting out from *A* and *B* respectively, at any instant, will reach any point in PP' in the same time, since the paths are exactly equal. It is also clear that each wave

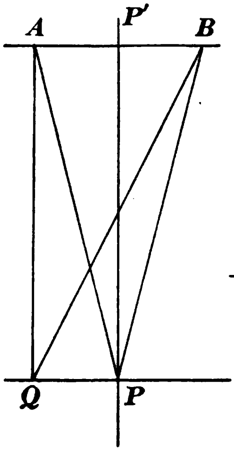

FIG. 60.

will, at any point, have a component of its motion normal to this plane, PP', and that these normal components will, at one instant, both be directed *toward* the plane, and a half period later they will both be directed *away* from the plane. In this plane, therefore, we have at every point condensation meeting condensation, and rarefaction meeting rarefaction, thus producing *double condensations and double rarefactions, in this plane* PP'. This plane is therefore a locus of maximum *change in density*, and is consequently a locus of *maximum intensity of sound as perceived by the ear.*

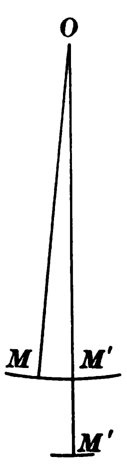

FIG. 61.

It is also to be noted that two air particles symmetrically situated on either side of this plane, although moving in opposite directions, actually meet *in the same phase*, since their displacements from their respective positions of rest are equal and have been produced *in the same time.*

In any other plane drawn through AB, the *condensations* and *rarefactions*, and consequently the *intensity*, will present a series of fixed maxima and minima. If we consider the disturbance from the single point O (Fig. 61), along two lines OM and OM' very near to each other, so that we may assume that the motions have experienced the same conditions in passing from O to M and M', then we shall find the particles in M and M' to be in the *same phase of vibration* if the difference of the distances OM and OM' be an even number of half wave lengths. They will be in *opposite phase* if the difference of the paths OM and OM' be an *odd* number of half wave lengths. In the first case the velocity and displacement of the two particles at M and M', *due to the single source at* O, will be equal and in the same direction; in the second, they will be equal and in opposite directions.

From the foregoing considerations we see that when sound waves from two identical sources A and B (Fig. 60) meet at a point Q, we shall have *maximum* change in *density* if

$$AQ - BQ = 2n\lambda/2 \qquad (205)$$

and *minimum* change in *density* if

$$AQ - BQ = (2n + 1)\lambda/2 \qquad (206)$$

that is, according as the difference of path $AQ - BQ$ is an *even* or an *odd* multiple of $\lambda/2$.

Hence for maximum intensity the difference in path is an *even* number of half wave lengths; for minimum intensity the difference in path must equal an *odd* number of half wave lengths. In this way two equal sounds may be added together so that at certain points in space they will mutually reënforce each other, and the sound will be very loud; in other points where the vibrations meet in opposite phase, one sound added to another may produce silence.

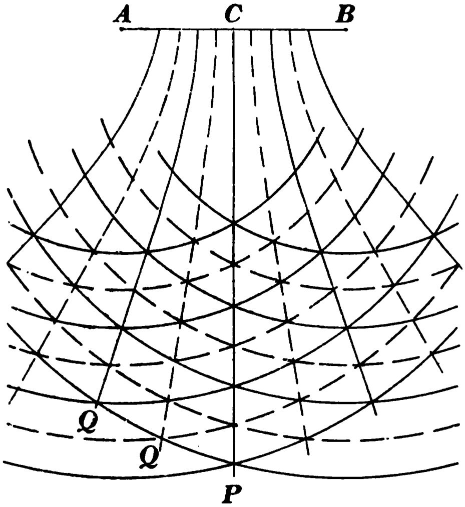

FIG. 62.

121. Curves of Maximum and Minimum Intensity. The interference of waves from two identical sources of sound may be shown graphically by the following construction: About the point sources A and B (Fig. 62) describe arcs of circles with equal radii, where the full line arcs denote condensations and the dotted arcs represent rarefactions. The shortest distance between any two concentric arcs of the same kind is therefore some multiple of the distance from one condensation to another condensation, or from one rarefaction to another, *i.e.* some multiple *of a wave length* λ, of the sound in question. Similarly, the distance between a full line arc and the next adjacent dotted arc described about the same center is *a half wave length.*

From the foregoing considerations it is evident that at all points on the line CP, normal to AB at its middle point, the two sets of waves *meet in the same phase*, condensation from A meeting condensation from B, and rarefaction from A meeting rarefaction from B, as indicated by the intersection of the corresponding arcs. The full line CP is therefore *a line of maximum intensity*, and the remaining full lines cutting AB, being also loci of particles in the same phase of vibration, are *also lines of maximum intensity*. The dotted lines on either side of CP, drawn through the intersections of rarefactions from A with condensations from B, or *vice versa*, are consequently lines of *minimum intensity of sound*. These curves are all hyperbolae, defined by the equation

$$AQ - BQ = \pm n\lambda/2$$

where Q is any point whatever, and n is any integer 0, 1, 2, 3, etc. The lines of maximum intensity are given by the even values of n, and those of minimum intensity by the odd values.

We should remember that there is *no loss of energy* due to interference. At the maxima the amplitude is *double*, and the energy *four times* that due to a single source. At the minima, the amplitude and the energy are both zero. Hence the average energy over the surface is *twice* that due to a single source, as it should be.

122. Experiments illustrating Interference.

(*a*) *Sources Identical*

If a tuning fork be sounded and rotated slowly before the ear, an intermittent or pulsating sound will be heard. At four certain positions, it will be found that the sound of the fork seems to disappear almost entirely, only to reappear again in force on moving the fork. In this case the two identical sources are the two prongs of the fork, which are vibrating in opposite directions as indicated in the arrows (Fig. 63). As the prongs approach, there is a rarefaction on the outside of each prong at f and g, and a double condensation on the inside, starting out toward d and e. These two sets of disturbances differing in phase by half a period meet along the dotted lines and produce a minimum sound at all points in these lines. Outside these lines the sound may be heard as usual.

To show that this is a true case of interference the fork may be held over a resonator which responds loudly, and rotated till the sound falls to a minimum. On slipping a small cylinder of paper over one prong of the fork, so as to cut off one set of waves, the sound reappears, but disappears again if the paper be removed.

If a large fork or organ pipe be sounded in a large room and an observer walk about the room, certain places will be found where the sound is uncomfortably loud while at others almost no sound is heard. In this case interference occurs between the direct waves from the fork and the systems of waves reflected from the sides of the room.

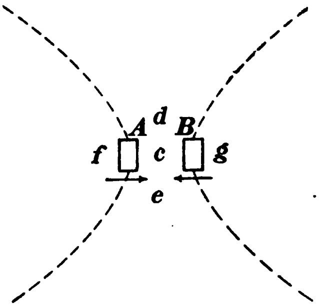

FIG. 63.

(*b*) *Sources not Identical. — Beats*

If two tuning forks, tuned to unison and furnished with resonance cases, be sounded together, a full even tone is heard. If now the prongs of one fork be weighted with wax, so as to decrease slightly the frequency of that fork, we shall find that when sounded together, the forks give out a throbbing or beating tone. If the pellets of wax be made larger, the number of beats per second increases.

In this case the phase difference $\lambda/2$ is not due to the difference in path traversed by the waves from the sources, as hitherto, but has resulted from a difference in the *frequencies* of the two forks. If the forks make respectively 100 and 101 vibrations per second, then the second fork gains one vibration upon the first every second. Under these conditions, if the two forks *started* together in the same phase, they would be in opposite phase at the end of half a second, and in coincidence of phase again in another half second. From this it appears that two forks whose frequencies are m and n vibrations per second will make $m - n$ beats per second.

Two organ pipes of the same pitch, when mounted upon a wind chest, will give loud beats if, while they are sounded, a card be slipped slightly over the lip or the end of one of the pipes. A tuning fork mounted upon a resonance case and sounded gives distinct beats if carried rapidly *toward* or *away* from a reflecting wall. The experiment may be rendered more striking by swinging the fork as a pendulum, while sounding, at a short distance from the wall.

MUSICAL RELATIONS

CHAPTER XIV

MUSICAL SCALES

123. Pitch. The pitch of a sound depends upon the vibration frequency of the sounding body. When the number of vibrations per second is great, the pitch of the tone is high or *acute*; when small, the pitch is low or *grave*. If two sounds are produced by the same number of vibrations per second, they are said to have the same pitch, or if sounded together, they are said to be in *unison*. *Musical sounds* are those which produce a pleasing effect upon the ear and *have a definite pitch.* A noise is a confused mass of sonorous vibration in which the ear is unable to detect any definite pitch. In a musical sense, the pitch of a sound may also refer to the relative position of the sound upon some arbitrary scale of reference adopted by musicians.

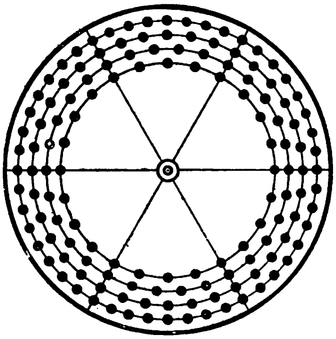

FIG. 64.

The pitch of a sound may be determined by means of an instrument called the *siren* (Fig. 64). The siren consists of a pasteboard or metal disk, bearing on its circumference a series of concentric circular rows of equidistant holes about 3 mm in diameter, and mounted on an axis which can be rapidly revolved in front of a nozzle delivering air from a blower. When an opening comes in front of the nozzle the air rushes through, forming a condensation, followed by a rarefaction during the interval in which the air is cut off. In this way is formed a series of regular puffs, which gradually blend into a low musical tone, whose pitch rises as the speed of rotation of the disk increases. The siren tone may be tuned to that of the given note, and its frequency determined from the angular velocity of the disk and the number of holes passing the nozzle in one revolution.

124. Musical Intervals. When two tones are sounded together, the ear recognizes a certain relationship, or want of relationship, between them, dependent upon their relative pitch, but entirely aside from their absolute vibration frequencies. This relationship is termed a *musical interval*, and is expressed as a simple ratio between the vibration frequencies of the two tones in question.

A number of these ratios have specific names in musical nomenclature, arising for the most part from the number of the note in the series. The interval between two notes whose vibration frequencies are in the ratio 1/1 is called *unison;* 2/1, an *octave;* 3/1, a *twelfth;* 4/1, the *double octave;* 3/2, a *fifth;* 4/3, a *fourth;* 5/4, a *major third;* 6/5, a *minor third;* 5/3, a *major sixth;* 8/5, a *minor sixth.*

This completes the list of so-called *consonant intervals*, although the list may be and probably will be extended in the course of time. It is interesting to note that the third, both major and minor, were originally classed among the dissonant intervals, and the minor third was not regularly used until the middle of the eighteenth century.

125. The Diatonic Scale. The rule for consonant intervals extends to combinations of several sounds. In order that three or more tones when sounded together may be concordant, it is necessary that their respective intervals not only with the fundamental, but also with each other, should be expressed by simple ratios. Thus when we sound together three notes whose frequencies are as $4:5:6$, there is produced a pleasing effect. This combination of three tones is called a *major triad.* The diatonic scale is built upon three sets of such triads.

The notes of the scale are indicated by the letters C, D, E, F, G, A, B, c. These letters may represent the frequencies of the various notes as well.

In the key of C, the three major triads are

$$\left.\begin{array}{ll}\textit{Tonic} & C:E:G\\ \textit{Dominant} & G:B:d\\ \textit{Subdominant} & F:A:c\end{array}\right\} 4:5:6$$

The vibration ratio in terms of C, the fundamental, are obtained as follows:

$$\frac{E}{C}=\frac{5}{4} \text{ or } E=\frac{5}{4}C$$

$$\frac{G}{C}=\frac{6}{4} \text{ or } G=\frac{3}{2}C$$

$$\frac{B}{G}=\frac{5}{4} \text{ or } B=\frac{5}{4}G=\frac{5}{4}\cdot\frac{3}{2}C=\frac{15}{8}C$$

In this way, the frequencies of the entire seven notes may be expressed in terms of the fundamental C, the octave c being set equal to $2C$, and we have the following relations between the various notes of the scale:

Frequency	72	81	90	96	108	120	135	144
Name	*C*	*D*	*E*	*F*	*G*	*A*	*B*	*c*
Syllable	*Do*	*Re*	*Mi*	*Fa*	*Sol*	*La*	*Si*	*Do*
Ratio	$\frac{1}{1}$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Intervals		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

The fractions termed *intervals* are obtained by dividing each ratio by that of the note immediately below it.

From this it appears that in the perfect diatonic scale there are three different intervals, 9/8, 10/9, 16/15. The first two intervals are termed *whole tones*, and the last a *half tone*.

The *minor triad* is composed of three notes, whose frequencies are 10 : 12 : 15, and a *minor scale* similar to the *major scale* may be built upon three of such triads. The vibration ratios for the various notes in terms of the fundamental may be obtained in the same way. On making the computations, we have

Frequency	72		81		.86.4		96		108		115.2		129.6		144
Name	*C*		*D*		*E*		*F*		*G*		*A*		*B*		*c*
Ratio	$\frac{1}{1}$		$\frac{9}{8}$		$\frac{6}{5}$		$\frac{4}{3}$		$\frac{3}{2}$		$\frac{8}{5}$		$\frac{9}{5}$		2
	La		Si		[illegible]		Re		Mi		Fa		Sol		La
Intervals		$\frac{9}{8}$		$\frac{16}{15}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$	

It will be found that three additional notes will be needed, viz., three notes below *E*, *B* and *A*, in order to produce the minor scale.

126. Transposition. In order to accommodate different voices or instruments, it is frequently desirable to change the keynote of the scale from *C* to some other note in the scale. The vibration ratios would then have to be applied to the *new keynote* as a fundamental, and the corresponding frequencies for the several notes computed. If it were desired to begin the scale with *D*, then, on computing the frequencies for the scale, it would be found that, beside the keynote *D* and its octave *d*, the *G* and *B* were right, and that the *A* and *E* differed but slightly from the required frequency, but the notes *F* and *c* would be found to be too low in each case. This must be remedied by the introduction of two new notes, *F* sharp and *c* sharp, in order to sing or play in the key of *D*. These two sharps are introduced at the beginning of the staff, and form the signature of the key.

127. The Tempered Scale. Since each change of key entails the introduction of new notes, both for major and minor scales, it is apparent that the number of notes demanded for each octave, in order to render a piece of music in any key, would be very greatly increased, so much so, indeed, that in the case of an instrument of fixed tones, as the piano or organ, it becomes practically impossible to manipulate so many keys. On this account, a compromise system, known as the system of *equal temperament*, has been adopted. In this system the whole tones are made all alike, and the half tones are half the whole tones. In other words, *there is an equal interval between each pair of consecutive notes.* There are thus added five new tones to the octave, making thirteen tones in all. The common ratio between the frequencies of any tone and the tone next above it is the *twelfth root of 2*, or 1.059. In any instrument tuned to this system, the only accurate intervals are the octaves, all the others being slightly false. The fifths are slightly flat, and the thirds are too sharp. Music rendered in this system is considered inferior to that played in just intonation. Trained singers, and performers upon instruments like the violin or slide trombone, are free from the limitations of the system of equal temperament, and in many cases approximate closely the intonation of the diatonic scale.

RESONANCE PHENOMENA

CHAPTER XV

VIBRATORY PHENOMENA AND RESONANCE

128. Composition of Vibrations at Right Angles. In the study of vibratory motion, some curious and beautiful results are obtained from combining two simple harmonic motions at right angles to each other. Owing to the rapid motion of sounding bodies the eye is unable to follow them, and some special device is necessary. In Fig. 65 are shown two tuning forks *L* and *M*, each furnished with plane mirrors, one set to vibrate vertically, the other horizontally. If now a beam of light from *R* be allowed to fall upon the mirrors successively and be reflected to the screen *S*, when the fork *L* is set vibrating, the spot is seen drawn out into a vertical band. Similarly, if the

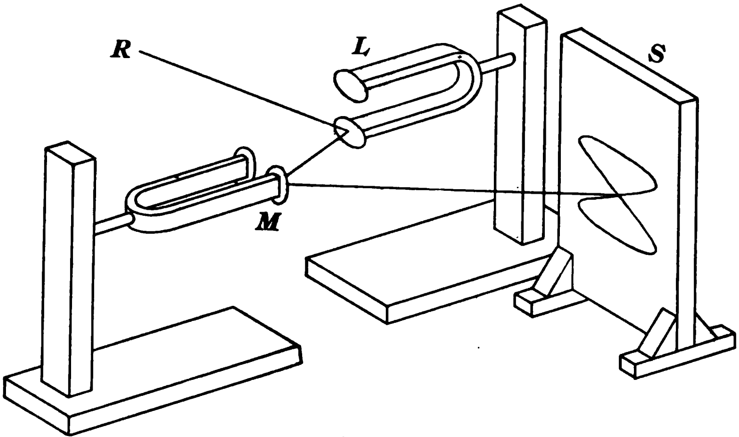

Fig. 65.

fork *M* be set in motion and *L* kept at rest, the spot is drawn out into a horizontal band. If the two forks be vibrated at the same time, the spot is made to follow the motion of the two vibrating systems and traces some form of what is known as a Lissajous' curve, of which various forms are shown in Fig. 66.

If the frequencies of the two forks be in the ratio of 1:1, then the characteristic figure will be an ellipse, having for its special forms the two straight lines. If the tuning of the forks be exact, the figure is motionless upon the screen and gradually decreases in size as the amplitude of the vibrations of the forks sinks to zero. In most cases, however, the tuning is only approximate, and the figure takes the successive forms indicated (Fig. 66), passing from left to right and back again. When the figure has run through the complete cycle, we know that one of the forks has gained or lost one complete vibration as compared with the other. We have thus a method of observing beats optically, and of determining the relative frequencies of vibrating bodies with great precision.

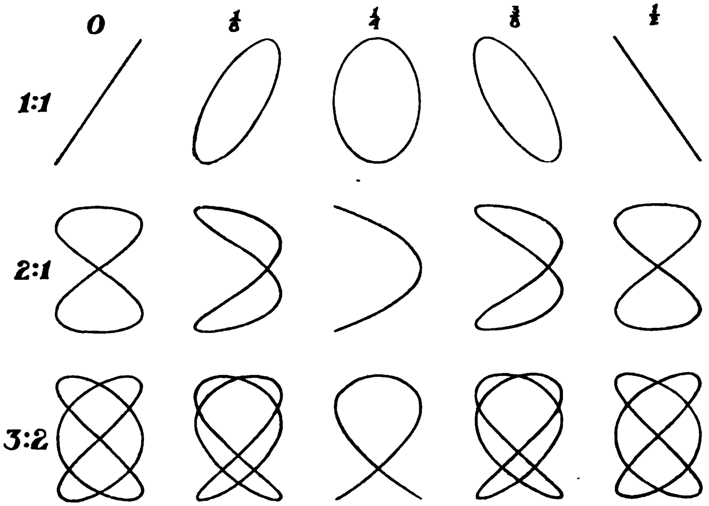

FIG. 66.

If, however, the frequency of fork *M* be twice that of fork *L*, we shall have the curve shown in Fig. 65, of which different forms appear in Fig. 66. In this figure are also shown the curves for the interval 3:2. The curves are drawn for the phase differences indicated at the top, where these differences are stated with respect to the component having the higher frequency.

These phase differences are exact at the beginning and close of any complete period. Thus in the ratio 3:2 the x motion has the higher frequency, 3, while the y motion has a frequency of 2. At the beginning the x motion leads in phase by $0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}$ or $\frac{1}{2}\,T$, as the case may be, and also when the motion on the x-axis has made *three* vibrations, and that on the y-axis has made *two*.

This experiment was first described by the French physicist Lissajous, in 1857, and the curves are known as Lissajous' curves. The method is applicable to the study of any vibrating system upon which a bright point as a minute globule of mercury can be fixed, while the fork with which the system is to be compared is armed with a lens of low power through which the mercury globule may be viewed by a microscope.

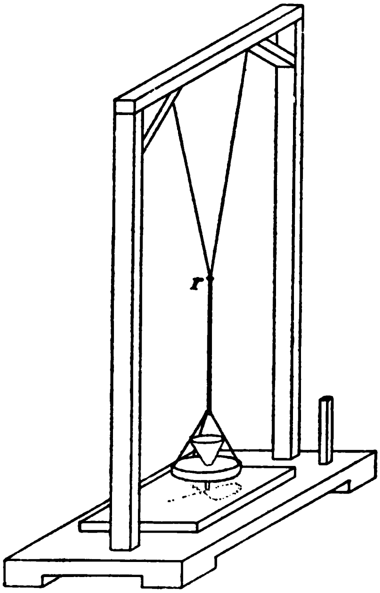

FIG. 67.

The same figures may be obtained by means of the Blackburn's pendulum, shown in Fig. 67. In this apparatus a heavy lead disk carries a funnel filled with sand, ink, or other material for leaving a trace of the motion upon a prepared paper beneath it. The disk is hung from two cords about one meter in length, and over the two cords is slipped a small ring *r*, by means of which the system is divided into two pendulums of different period, hung from the same support. On setting the disk vibrating it traces the figures characteristic of the ratios represented by the square roots of the lengths of the entire pendulum and the part below the ring.

129. Graphical Method for Lissajous' Figures. Draw two concentric circles (Fig. 68), with radii proportional to the amplitudes of the two harmonic motions, and through their common center *O* draw the rectangular diameters *AB*, *CD*.

Divide each quadrant of both circles into the same number of equal parts; some multiple of four is usually most convenient. Through the points of division of the circle *AB* draw lines parallel to *CD*, and through the divisions of *CD* draw lines parallel to *AB*. The resulting rectangle will contain all the figures arising from any possible combination of two simple harmonic motions of commensurable periods; and the curves will, in general, be tangent to the sides of the rectangle.

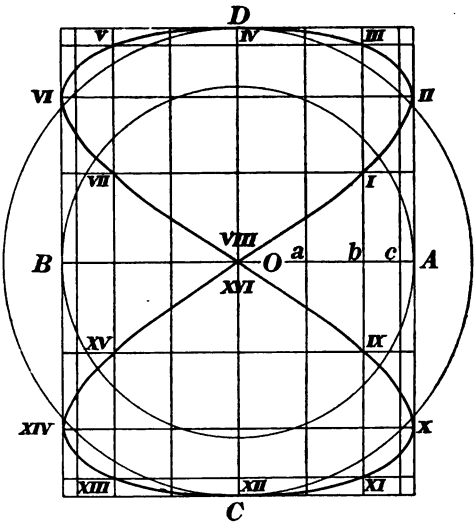

Fig. 68.

The center of the circles corresponds to a phase difference of zero between the two components, that is, to $\delta = 0$; and it is taken as the starting point for tracing all curves of phase difference zero or $T/2$.

If, as in Fig. 68, the circles have been divided into sixteen equal parts, then each point of intersection on the diameter *AB* corresponds to a phase difference of $T/16$; that is, to one sixteenth of a period. Hence if we start to trace a curve from *a* in the figure instead of from *O*, we shall produce the curve corresponding to a phase difference of *T*/16. This means that at the instant when the *y* component passes through *AB* in a positive direction, and the *y* displacement is therefore zero, the *x* component has already reached *a* in the positive direction, or is in advance of the *y* component by *Oa* or *T*/16. In like manner, *b* corresponds to a phase difference of *T*/8, *c* to 3 *T*/16, and *A* to *T*/4. Returning toward *O*, it will be seen that *c* also corresponds to a difference of phase of 5 *T*/16, *b* to 3 *T*/8, *a* to 7 *T*/16, and *O* to *T*/2, with larger values for points to the left of *O*.

Suppose now that we wish to trace the curve corresponding to the vibration frequencies one to two; two for the horizontal and one for the vertical component, and with no difference of phase. Starting from *O*, we count two points horizontally to the right and one up and reach *I*; again two to the right and one up for point *II*, and so continue, numbering the points in order until we pass through the starting point in the same direction as at first, being careful always to complete the motion in one direction before beginning the retrograde motion. An excellent check upon the accuracy of the location of the points is found in the fact that points equidistant from the axis of symmetry *AB* differ in number by eight in every case; that is, by half a vibration.

If now a smooth curve be traced through the points in order, we see that the moving point, being subject to both motions, describes two spaces horizontally and one vertically in the same interval of time, and consequently passes through the corners of rectangles two spaces long and one space high in every case. The spaces themselves increase or decrease according to the simple harmonic law. Great diversity of figure may thus be obtained with successive differences of phase between the two component motions.

To combine two motions of frequencies two to three, we should simply count three spaces in one direction and two in the other and proceed in other respects as already described.

130. Free and Forced Vibrations. If a system when displaced from its position of equilibrium is urged to return to that position by forces, either internal or external, which vary directly as the displacement, it will, when freed from the disturbing force, execute simple harmonic vibrations about its position of equilibrium as a center. The *period* of such a vibration *depends upon the nature of the system*, and is independent of either the amplitude of vibration, or of the forces tending to oppose the vibration, provided each be small. Such a vibration is called a *free vibration*, and the period is termed the *free or natural period* of the system.

In the ideal case of no friction, a body once started would continue to vibrate forever, since there would be nothing to stop it. In nature, however, all vibrations are checked with more or less promptness by means of opposing forces which may be designated under the general name of friction. In such cases the amplitude gradually decreases to zero, while the period remains constant, and is independent of the friction, provided it be small. Such a vibration is termed a *damped vibration.* Examples of *free vibration* are seen in the motion of a simple pendulum, of a guitar string or of a tuning fork when bowed and allowed to swing freely.

When a system that is free to vibrate is subjected to the action of a periodic force that varies as an harmonic function of the time, we have the conditions necessary for a *forced vibration.* The ensuing motion is the response of the system to the impressed, external force, and continues so long as the force continues. Examples of *forced* vibration are seen in the motion of the pendulum of a clock or the balance wheel of a watch, in the vibration of a tuning fork driven by an electric current, or of the sounding board of a piano or body of a violin. Since any free vibration is always more or less damped, and therefore soon sinks to zero, it follows that any *maintained* vibration is a *forced* vibration; the motion of the vibrating system being maintained by an external impressed force which varies with the time. The *period of the forced vibration* is the *period of the force*, and the amplitude is proportional to the force.

The characteristics of *free* and *forced* vibrations may be contrasted as follows: A *free* vibration gradually *dies away* on account of frictional forces. A *forced* vibration *is maintained* so long as the impressed force continues, and when it ceases the free vibration ensues and gradually sinks to zero. The *period of a forced vibration* is the *period of the impressed force*, while the period of a *free vibration* depends upon the *constitution of the system*, and is entirely independent of the forces causing it, so long as the amplitude is small.

131. Resonance. A special case of *forced vibration* is that in which the period of the impressed force coincides with the free period of the system. In such a case the system rapidly absorbs energy from the individual, periodic pulses, and soon vibrates with large amplitude. Theoretically it is due to the friction alone that the amplitude does not become infinite. *It thus appears that a system free to execute vibrations of a definite period is capable of selecting and absorbing from the surrounding medium energy in the form of vibrations of the same period as those which it can execute.*

This is known as the *principle of resonance*, which finds its applications in every department of physics. Resonance depends upon the cumulative effect of small impulses applied to a system at exactly the proper time to produce the maximum effect.

132. Illustrations of Resonance. Two strings stretched upon a sonometer, if turned to unison, will mutually transmit vibratory motion, by means of synchronous impulses sent through the air and through their common support. If either string be set in vibration, the other begins to vibrate.

Let two heavy pendulums of the same period be mounted upon a wooden frame which yields slightly to their motion, and let one be set vibrating while the other remains at rest. In a few minutes it will be seen that the second pendulum is acquiring vibratory motion through the support. Its motion gradually increases until the two are swinging with equal amplitude, but with a *phase difference of half a period.* The second pendulum continues to lag behind the first, gradually absorbing its energy until the first is brought to rest, after which the phenomenon is repeated in the reverse order.

If a tuning fork be held over the mouth of a tall jar (Fig. 69) partly filled with water, it will be found, on pouring in more water, that for a certain length of air column the sound of the fork is powerfully reënforced. If the fork be removed and a blast of air from a flat tube be blown across the top of the jar, the sound produced will be in unison with that emitted by the fork. The cylinder thus behaves as a stopped organ pipe (Art. 140) and the air column is very nearly equal to one fourth the wave length of the sound produced by the fork. The hollow wooden cases used to support tuning forks are in reality closed or open pipes tuned to reënforce the tone of the fork.

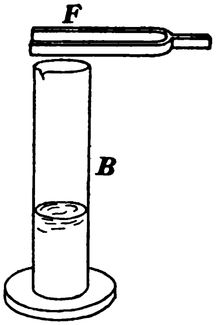

FIG. 69.

Let two tuning forks mounted upon suitable resonance cases and accurately tuned to the same pitch, be placed at opposite ends of a room. If one be bowed and then quieted, it will be found that the other is sounding audibly. Accurate tuning is necessary for success in this experiment. If a number of forks of different pitch be sounded together, the second fork responds to none but the one of its own frequency. Obviously the fork can absorb from the air only those wave lengths of sound which it itself can emit.

A heavy bridge is often set to vibrating vigorously by the footfalls of a small dog trotting across it. Soldiers when crossing a bridge are commanded to break step to avoid the possibility of synchronous vibration of the bridge.

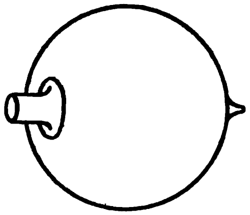

FIG. 70.

The resonators of von Helmholtz (Fig. 70) consist of hollow spheres of brass, furnished with a tubular opening for the reception of the sound wave, and opposite it a small conical tube to be inserted into the ear.

The free period of the enclosed mass of air determines the pitch of the tone to which the resonator will respond. To all other tones it remains practically silent. By means of a series of such resonators von Helmholtz was enabled to pick out the various overtones in the note of a piano string, and thereby to *analyze* a sound into its constituent tones.

133. Stationary Vibrations. Suppose one end of a long flexible cord be fixed and the other end be moved quickly up and down by the hand in a vertical plane. For each up and down motion of the hand a single pulse will run the length of the cord, be reflected at the fixed end and retrace the length of the cord, to be reflected again at the hand. In each case the reflection of the pulse in the cord will involve a *change of sign* both in the motion and in the nature of the disturbance itself, since a depression in the cord is returned as an elevation, and an elevation as a depression. After two reflections, therefore, the disturbance will have traversed the length of the cord twice and will be identical both in *direction* and *kind* with the original disturbance, and may be regarded as starting out anew.

If now the hand be maintained in simple harmonic motion, a series of harmonic waves will run along the cord and be reflected at the fixed end as before. Each wave after two reflections will coincide both in *direction* and *kind* with the new outgoing wave, if the time required to travel twice the length of the cord be some *whole* number, k, times the period of the motion maintained by the hand. In other words, if

$$\frac{2l}{V} = kT \tag{207}$$

where l is the length of the cord, V the velocity of the pulse along the cord, T the period of the motion, and k is *any integer* as 1, 2, 3, 4, etc.

Under the above condition it is clear that any disturbance however small will soon be increased sufficiently to set the cord into vibrations of wide amplitude at all points where the direct and reflected waves *coincide in phase.* Such points are called *antinodes.* At certain other points, however, the incoming and outgoing waves meet in *opposite phase* and produce points of *minimum motion;* such points are called *nodes*. As a result of the superposition of the direct and reflected waves the cord is broken up into a series of vibrating loops, or ventral segments, (Fig. 71), separated by points of minimum motion *N*.

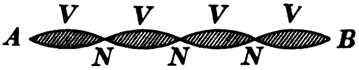

FIG. 71.

Such a vibratory motion is called a *stationary vibration* or a *stationary wave*. The distance from one node to the next, or from one antinode to the next, is *one half wave length of the pulse in the cord*. Stationary vibrations may thus be set up in any medium capable of transmitting wave motion, and the phenomena of nodes and antinodes developed according to the principles just laid down. In all cases, the distance from *node to node*, or from *antinode to antinode*, is $\lambda/2$, *for the medium in question*. From *node* to *antinode* is $\lambda/4$.

In all cases of sustained tones, as those from organ pipes, tuning forks, piano, violin, or guitar strings, the vibrating medium, whether air column, bar, or string, is executing *stationary vibrations*, and consequently presents the characteristic feature of stationary waves, *i.e.*, *nodes* and *antinodes*.

134. Laws of Transverse Vibrations of Strings. A string fastened at the ends and vibrated transversely executes stationary vibrations as described in the previous article. The vibration of the string gives rise to a note of definite pitch, dependent upon the physical constants of the string and upon its mode of vibration. The condition for stationary vibration is

$$\frac{2\,l}{V} = kT = \frac{k}{n} \tag{208}$$

where n is the frequency of the note produced by the string. Of the various modes of vibration dependent upon the value of k, the simplest is that in which the string vibrates as a whole, with a node at each end. In this case $k = 1$, and

$$n = \frac{V}{2\,l} \tag{209}$$

This vibration is called the *fundamental vibration*, and the tone the *fundamental* or lowest tone given by the string.

It may be shown mathematically that the velocity of a ***transverse wave*** in a thin, flexible string, of density d and radius r, when stretched by a *force* of T dynes, is given by the expression

$$V = \sqrt{\frac{T}{\pi r^2 d}} \tag{210}$$

If we substitute this value for the velocity in the equation for the frequency, we have

$$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}} \tag{211}$$

Hence the frequency of the fundamental tone emitted by a string vibrating transversely, varies

(*a*) Inversely as the length of the string.

(*b*) Inversely as the radius of the string.

(*c*) Inversely as the square root of the density of the string.

(*d*) Directly as the square root of the stretching force.

These laws are deduced theoretically from the case of a long, thin and perfectly flexible string, and are very nearly realized in the case of silk or gut strings. If metallic strings are used, the rigidity of the string acts as an added stretching force, thus making the frequency higher than the formula would indicate.

135. Melde's Experiment. If a thin, flexible cord some four meters long be attached to some convenient source of simple harmonic motion, and the stretching force be properly adjusted, the laws of vibrating strings may be verified by experiment.

In Melde's experiment one end of the cord is attached to one prong of a vertical tuning fork, and the other end carrying a pan is passed over a light pulley. If the prongs of the fork stand normally to the cord, and vibrate in the direction of its length, then the end of the cord will be displaced longitudinally at each vibration of the fork. At the forward swing the cord relaxes and drops down, rises and stretches tight on the backward swing of the fork, passes the position of rest and rises above on the next forward swing, falls to the middle on the backward swing and to its lowest position again on the forward swing. The fork has thus made *two complete* vibrations, for a *single vibration of the cord*, or the cord in this position, *vibrates an octave lower than the fork.*

If the stretching force be properly adjusted by weights placed in the pan, the cord opens out into a wide spindle which remains fixed while the vibration continues. If the stretching force be reduced to one fourth its value, the velocity of the pulse will be one half its previous value, and the cord will now present two spindles with a node in the center. One ninth the original stretching force will give three spindles and two nodes. This verifies the law of the stretching forces.

If the fork be rotated about a vertical axis so that its vibrations are normal to the length of the cord, the cord will vibrate in unison with the fork, and for the original stretching force, it will divide into *two spindles*, where it previously vibrated in *one*. Since *each half* now vibrates in unison with the fork, while before the whole cord vibrated an octave *below* the fork, the law of lengths is demonstrated.

136. Segmental Vibration. It has been shown that a string may execute stationary vibrations under the condition that

$$\frac{2l}{V} = kT = \frac{k}{n}$$

This means that the frequency of the tone emitted by a string may depend upon *its mode of vibration*, as well as upon its *length*, *diameter*, *density*, or the stretching force to which it is subjected. In the case of a string sounding its fundamental, k is unity and the string *vibrates as a whole*, with nodes at the two ends. This is the simplest mode of vibration. The next simplest is when the string is divided into two segments with a node in the middle. In this case $k = 2$, and the frequency is double that of the fundamental. This tone may be drawn from a string by holding it at its middle point with a thin shaving of cork and bowing it lightly at about one ninth its length from one end. Here the string may be considered as made up of two strings of equal length vibrating in unison, and producing a tone an octave above the fundamental. In a similar way the string may be broken up into three, four, five, six, or any number of equal segments, corresponding to the integral values of k. The frequency of the tone in each case is inversely as the length of the segments, and consequently if the string vibrate in three segments, the frequency is *three times* that of *the fundamental;* if it vibrate in fourths, the frequency is *four times* the fundamental; if in fifths, five times, etc.

The experimental demonstration of segmental vibration is most readily accomplished by means of a piece of piano wire about four meters long, tightly stretched between two bridges clamped to the top of a long table. One end should be attached to a key or screw in order to vary the stretching force at will. A series of stiff paper markers should indicate the aliquot parts of the string, as the thirds, fourths, sixths, etc. A thin shaving of cork should be slit and fixed upon the wire so as to slide freely upon it. Little riders of white and colored paper may be distributed along the wire, with the white ones at the aliquot ~~parts.~~ points of division.

If the cork be held at one eighth the length of the wire and the bow be applied, gently at first, then more vigorously, the riders at the eighths will remain seated while all the others will be thrown off. The frequency of the tone emitted by the string will be eight times that of the fundamental. In a similar way the string may be made to vibrate in sixths, fifths, fourths, thirds and halves, the position of the cork marking a node in each case. If the cork be *not* upon some aliquot point, no satisfactory vibration and no note of definite pitch can be produced, hence *we conclude that a uniform string may vibrate as a whole, or in any number of equal parts, and the frequency of the note emitted will be proportional to the number of parts.*

137. Overtones. If the piano wire of the preceding experiment be vigorously bowed and then damped at one fourth its length, the note will be observed to change its character. The fundamental will disappear and the second octave, a note whose frequency is four times that of the fundamental, will be heard instead. This shows that while the string was *vibrating as a whole*, it was also *vibrating in fourths*, and further, that the note emitted was made up of the *fundamental tone and the second octave*. By successively bowing the string, and damping it at the middle, and at one third its length, the first octave and the twelfth are found to be present when the string vibrates freely.

It thus appears that a string may at the same time vibrate as a whole and divide into several sets of equal segments, thus giving rise to the fundamental and also to tones whose frequencies are much higher than that of the fundamental. The higher tones thus obtained are termed *overtones*, or *upper partials*. In case their frequencies are exact multiples of that of the fundamental, the entire series are called *harmonics*, in which the fundamental is properly termed the *first*, the first octave the *second*, the twelfth the *third harmonic*, and so on, since these tones represent a series in which the frequencies are as 1 : 2 : 3 : 4 : 5, etc.

If we consider the key of *C*, the first ten harmonics, counting the fundamental as the first, are C (1), c (2), g (3), c_1 (4), e_1 (5), g_1 (6), c_2 (8), d_2 (9), e_2 (10); where the *seventh harmonic* lies between a_1 and b_1 and may be represented by b_1 flat.

Of these tones it will be observed that the first six harmonics are consonant tones, and if sounded together, would produce the effect of a *perfect major chord*. In a string of uniform dimensions and homogeneous structure, the frequencies of the *upper partials* approach very nearly the exact relation demanded for harmonics. It is for this reason that the music from stringed instruments is so rich and pleasing.

Again, it is clear that if the string be bowed or struck at its *middle point*, that point cannot by any possibility be a node. Hence all partial tones which require the presence of a node at the middle of the string must of necessity be absent. The discordant effect of the seventh and ninth harmonics of a string are avoided in the case of a piano by having the hammer strike the wire at a distance of a little less than one seventh the length of the string from one end.

Problems

1. Show that the right and left hand members of the equation

$$V=\sqrt{\frac{e}{d}}$$

are of the same dimensions.

2. Assuming the velocity of sound in air at 0° C to be 331.36 m per second, calculate the velocity in hydrogen at the same temperature, having given the mass of one liter of hydrogen = 0.0896 g.

3. Find the temperature at which the velocity of sound in air is 356 m per second.

4. The flash of a hunter's gun is seen and after 5 sec the sound is heard. Required the distance from the observer to the hunter, the temperature being 22° C.

5. Colladon and Sturm measured the velocity of sound in the waters of Lake Geneva, and found that it traveled 1435 m per second, the temperature being 8°.1 C. Compute the coefficient of elasticity for water at this temperature.

6. A wire 50 cm in length and of mass 80 g is stretched so that it makes 100 complete vibrations per second. Compute the stretching force.

7. A string is attached to one prong of a tuning fork and after passing over a smooth peg is stretched with a force of 32×10^8 dynes. When the string is parallel to the motion of the fork it vibrates steadily in three segments. What stretching force is required to make it divide into two segments? Into five segments?

8. What stretching force is needed to have the above-mentioned string divide into eight segments, when the string stands at right angles to the motion of the fork?

9. Determine the vibration frequency of an air particle in a sound wave 10 m long ($t = 20°$ C).

10. If the first syllable of an echo reaches the ear 3 sec after the spoken word, how far distant is the reflecting surface? ($t = 20°$ C).

11. A stone is dropped into a well and the sound of the splash is heard after 5 sec. How deep is the well, if the temperature be 10° C?

12. An open organ pipe 120 cm in length is tuned correctly when the room temperature is 20° C. What will be the change in its frequency when the temperature rises to 32° C?

13. A workman strikes a blow with a hammer upon one end of an empty iron water pipe, 600 m long. A second workman placing one ear against the other end hears two sounds. How far apart are they in time? Temperature 20° C. $\left(V \text{ for sound in cast iron} = 5000\frac{\text{m}}{\text{sec}}.\right)$

14. A horizontal string carrying a small globule of mercury is viewed through a lens fastened to one prong of a tuning fork, placed at right angles to the string and vibrating horizontally. The fork has a frequency of 128. The fork is bowed and the string is tuned until the ellipse seen through the lens makes one complete rotation in 6 sec. If the stretching force be increased, the Lissajous' figure rotates faster. What is the per cent of error if this tuning be assumed as correct?

CHAPTER XVI

VIBRATION OF AIR IN PIPES AND CAVITIES

138. Vibration of Air Columns. In many musical instruments the vibrating body is a column of air in a pipe. Although the shape of the column and the mode of excitation may vary, yet the general principles of vibrating bodies will apply with but slight modification. When a series of similar pipes of the same diameter but of different lengths are sounded by blowing in turn across the ends of each, it will be found that the frequencies of the sounds produced are practically inversely as the length; that is, a slender pipe 10 cm long will give a note approximately one octave higher than a similar pipe 20 cm long, and two octaves higher than one 40 cm long.

If a tuning fork be held over a vertical pipe, the lower end of which is connected with a water supply for varying the length of the enclosed air column, it will be found that for a certain level of the water the air column in the pipe responds loudly to the vibrations of the fork. If pipes of different diameters are used, it will be found that under similar conditions the length of pipe responding to a given fork is nearly constant, diminishing slightly as the diameter increases. Again, if a closed pipe 20 cm long respond to a given fork, it will be found that an open pipe of the same diameter and same length will respond to a fork an octave higher than the first fork. This shows that *the pitch of an open pipe is an octave higher than that of a closed pipe of the same length.*

139. Length of Organ Pipe and Wave Length of Fundamental Tone. (*a*) *Open pipe.* Suppose an open pipe (Fig. 72), have placed before one end, a tuning fork or other suitable vibrator, which sends a series of sound waves into the pipe. Then *for* *the pipe and the tuning fork to vibrate in unison*, it is necessary for the reflected wave to return to the end B, in the proper phase to unite with the outgoing wave. This means that when the fork starts to swing from a'' to a' a condensation is sent into the pipe and runs the length BA while the prong moves the distance $a''a'$. At the open end A the condensation is reflected as a rarefaction (Art. 118), which starts into the tube at A at the same instant that a rarefaction enters at B, due to the backward motion of the prong from a'. These two rarefactions meet at the center of the tube, producing a *double rarefaction* for an instant, and pass on to the open ends to be reflected as condensations, at the instant the fork begins a second swing from left to right. The condensation at B unites with a new condensation from the fork, and the combined condensations run in at B while the reflected condensation enters at A. The two condensations meet at the middle, forming a *double condensation* at that point, just half a period later than the double rarefaction.

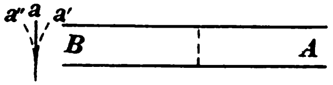

FIG. 72.

The disturbance is thus seen to travel the length of the pipe twice during one complete vibration of the fork, or, for the fundamental tone of the open pipe,

$$\frac{2l}{V} = T \qquad (212)$$

whence $2l = VT = \lambda$

This shows that for an open organ pipe *the wave length of the fundamental is twice the length of the pipe.*

(*b*) *Closed pipe.* In the closed pipe the fork on its swing from left to right sends in a condensation which runs to the closed end and being reflected as a condensation runs back to the open end, where it emerges and combines with the outgoing condensation caused by the fork on its swing from right to left. At the same time the emerging condensation is reflected *into* the pipe at the *open end* as a rarefaction which combines with the rarefaction left in the rear of the fork on its passage from right to left. The condensation or the rarefaction has in each case run the length of the pipe *twice* during *half* a vibration of the fork, or for unison with the fundamental,

$$\frac{4\,l}{V} = T \tag{213}$$

whence $$4\,l = VT = \lambda$$

That is, the wave length of the fundamental of a closed organ pipe is *four times the length of the pipe.* If we compare this result with that obtained for the open pipe, we see that the wave length of the closed pipe is double that of an open pipe of the same length, or the fundamental of a closed pipe is an octave lower than that of an open pipe of the same length.

140. Nodes in Open and Closed Organ Pipes. In the open pipe it was shown that a node existed in the middle, at which point there existed alternately double rarefactions and double condensations at intervals of half a period. A node therefore in a vibrating air column is to be considered *as a place of maximum change of density, but of minimum motion.* In an open pipe there is *always an antinode at each end,* since at the open end the motion is unrestricted. For the fundamental in an open pipe, therefore, there is a node in the middle and an antinode at each end, or the pipe contains *one half wave length* (Fig. 73 *A*).

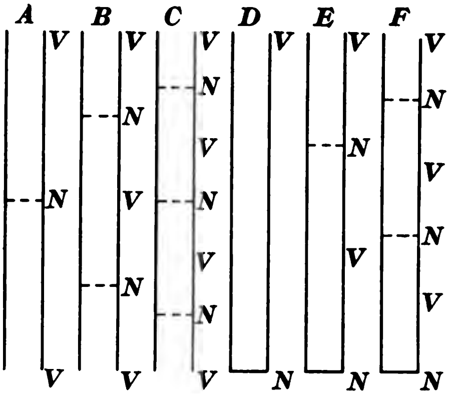

Fig. 73.

If the pipe be blown more strongly, it gives its first octave, the air column breaks up into segments, with a node at one quarter the length of the pipe from each end, and an antinode at the middle and at each end. In this case the pipe contains *two half wave lengths,* and the corresponding note is an octave above the fundamental (Fig. 73 *B*).

The second overtone is given by the air column forming three nodes, one at *one sixth* the length of the pipe from either end and at the middle, with antinodes between. In this case the pipe contains *three half wave lengths*, and the frequency of the tone is three times that of the fundamental (Fig. 73 *C*), and so on for higher tones. *Hence in an open pipe all the overtones are present.*

In a closed pipe, *there is always a node at the closed end*, since the air is at rest there, and as usual there is an antinode at the open end. For the fundamental the pipe contains *one fourth wave length* (Fig. 73 *D*).

The first overtone in the closed pipe is given by the air column forming a new node, at one third the distance from the open end. The pipe thus contains *three fourth wave lengths*, and the tone has three times the frequency of the fundamental (Fig. 73 *E*). The second overtone produces a node at *one fifth* and *three fifths* the length of the pipe from the open end. The pipe contains *five fourth wave lengths*, and the frequency of the tone is *five* times that of the fundamental (Fig. 73 *F*). *Hence in closed pipes only those overtones are present whose vibration frequencies correspond to the odd multiples of the fundamental.*

141. Quality of Sound. By means of his analysis of musical sounds von Helmholtz decided that the *quality* of a sound depends upon the number of overtones associated with the fundamental and upon their relative intensities, and is independent of their differences in phase. Quality of sound depends upon the form of the sound wave. In the ear the various constituents of a complex wave are separated and noted, and the effects of the various combinations distinguished. Von Helmholtz showed not only by direct analysis, but also by synthesis, that the sounds of certain musical instruments consist of definite overtones combined with the fundamental. By means of a series of tuning forks each of which gave a simple tone, he was able successfully to reproduce the notes of various musical instruments, and even to imitate most of the vowel sounds of the human voice.

An admirable instrument for the analysis of sound is found in the manometric capsule devised by Koenig. A cylindrical box (Fig. 74) is divided into separate, air-tight compartments by a flexible diaphragm, *D*, of thin rubber, or goldbeater's skin. Into one of the compartments, *A*, are introduced the sound waves by means of the funnel *M*. The compartment *B* contains illuminating gas, which enters through the tube *C*, and is ignited at the tip. If a condensation impinge upon the diaphragm, the gas in *B* is compressed and the flame leaps up; if a rarefaction enter *A*, the pressure in *B* is less and the flame is drawn down. If a musical note be sung into the funnel, the flame vibrates in unison with the air particles in *A*, and if it be viewed in a rotating mirror, the eye can determine at once the nature and constitution of the sound. In Fig. 75 the upper picture represents the appearance in the mirror when a simple tone is sounded in the funnel. The middle line represents the appearance of the flame when the octave of the first note is sounded, and the third shows the effect of combining the two. Manometric capsules may be attached to each of a series of resonators and the combination affords a means of instantly determining the composition of any note sounded in its vicinity.

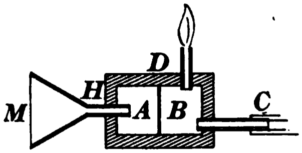

FIG. 74.

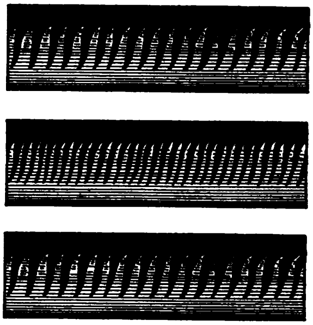

FIG. 75.

142. Kundt's Experiment.[1] The principle of resonance has been ingeniously applied by Kundt to the measurement of the velocity of sound in solids or in gases. A long glass tube (Fig. 76), some 6 cm in diameter, is furnished at one end with a loosely fitting piston, and has the other end closed by a sheet of thin rubber. A rod of brass or other metal is held by its

[1] For experimental details of Kundt's Experiment, see Manual, Exercise 34

middle point in a vise and one end is furnished with a disk of stiff paper which rests against the rubber membrane of the glass tube. The inside of the tube is dusted with fine cork filings or amorphous silica.

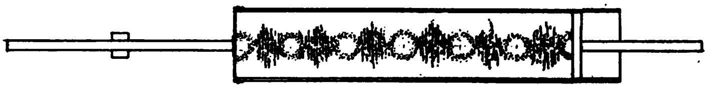

FIG. 76.

When the rod is stroked with a piece of chamois skin covered with a little rosin, it gives a loud clear note. It vibrates longitudinally, with a node in the center, after the manner of the air in an open organ pipe. The paper disk communicates the vibrations of the rod to the air in the tube, and when the length of the enclosed air column has been properly adjusted by means of the piston, the cork dust is tossed about into little heaps owing to the resonance of the air with the note emitted by the rod.

As we have already seen, the conditions of resonance demand that the path run over by the disturbance from the end of the rod to the end of the air column and back again, must be some even number of wave lengths, since the tube is closed at each end, and the motion is twice reflected with change of sign in direction, but without change of sign in condensation. When resonance has been established, the powder in the tube assumes the appearance shown in Fig. 76. The nodes, being points of minimum motion, are marked by small circles of the cork filings, while the antinodes are shown by transverse heaps of dust, where it has fallen at the cessation of the sound.

In Art. 133 it was shown that the distance from *node* to *node* is a half wave length of the disturbance in the medium. The nodal lengths in the air may be readily measured from the circles of powder, and the half wave length of the sound in air computed. Since the rod behaves as an open organ pipe sounding its fundamental, it follows that the length of the rod is *one half wave length of the sound in brass.* If V_a and V_b, λ_a and λ_b represent the velocities and wave lengths of sound in air and brass respectively, then, since the period is the same in each case, we have

$$\frac{V_b}{V_a}=\frac{\frac{\lambda_b}{2}}{\frac{\lambda_a}{2}} \tag{214}$$

or

$$V_b = V_a\frac{\lambda_b}{\lambda_a} \tag{215}$$

For gases, an additional tube with powder is fitted up and placed in contact with the other end of the rod. From the measured lengths of the nodal distances in the gas and in air the computation is made as given above.

143. Mouthpieces. The various forms of wind instruments differ chiefly in the mode of excitation of vibration of the enclosed column of air. That part of the instrument in which such vibration is excited is called the mouthpiece. Mouthpieces may be divided into three classes.

(*a*) Those in which the air is blown *across* a sharp edge or across an opening, as in the common tin whistle, the organ pipe, or the flute.

(*b*) Those in which the air is forced *through* an opening, either partially closed by an elastic tongue or reed which swings through, as in the common cabinet organ, harmonica, accordion, etc., or closed by a reed which shuts *down upon* the opening as in the clarionet, oboe and bassoon.

(*c*) Those in which the air is forced through a slit formed of two elastic membranes. This form of mouthpiece is made by cutting off the ends of a wooden tube obliquely on opposite sides and tying two strips of thin rubber over the faces so formed, so as to leave a narrow slit along their line of junction. If air be blown through the slit, a note will be produced, whose pitch will be modified by the body of air in the tube.

144. Vocal Organs. Of all musical instruments the larynx, the organ of human speech and song, is the most wonderful, both on account of its simplicity as well as for its extreme delicacy and range.

The larynx may be briefly described as a box formed by three plates of articulated cartilage which are moved by muscles, placed at the upper end of the trachea or windpipe.

The base of the larynx is formed of a large ring of cartilage called the *cricoid* (ring shaped) cartilage. Attached to this is the *thyroid* or shield-shaped cartilage, which is bent in the shape of a *V*, and fastened to the edges of the *cricoid* ring by its sides, the point of the *V* being turned to the front, forming the projection on the front of the throat known as "Adam's apple." At the back of the *cricoid* are fastened two small pointed cartilages, the *arytenoid* (funnel shaped) cartilages. Stretching between the *arytenoid* cartilages to the inner sides of the *V*-shaped *thyroid* are two elastic membranes, one fastened to each leg of the *V*. These are the *vocal chords*, *cc* (Fig. 77 *B*). Just above these are two folds of mucous membrane, known as the *false vocal chords*, *ff* (Fig. 77 *A*).

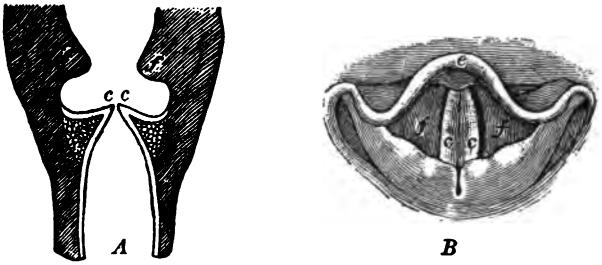

FIG. 77.

When the peak of the *thyroid* is not drawn down, the vocal chords are lax and the breath passes freely between them as in ordinary breathing. But when the peak of the thyroid is pulled down by the muscles attached to it, the vocal chords are stretched, the arytenoid cartilages move nearer to each other, and the thin, sharp edges of the vocal chords form a narrow slit across the windpipe, through which the air is forced, causing them to vibrate as the rubber membranes in the third form of mouthpiece described in Art. 143.

The pitch of the tone produced by the vocal chords depends upon their size, length and tension. The tension is controlled by the attached muscles, so that the singer can vary at will the pitch of the tone produced.

The quality of the sound produced is modified by the resonant qualities of the cavities of the mouth, throat and nasal passages.

145. The Ear. The human ear consists of three well-marked divisions, termed the *external*, *middle* and *internal ear*. The external part includes the familar appendage at the side of the head, and the auditory canal, or *meatus*, *M* (Fig. 78), in which is shown a section through the right ear.

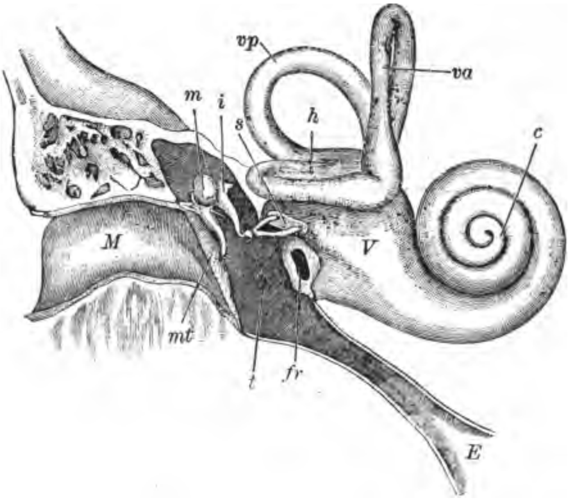

FIG. 78.

The meatus is closed by the *tympanum*, or eardrum, *mt*. This separates the external from the middle ear, and acts as a receiving membrane against which the sound waves impinge.

In the middle ear is found a chain of three small bones: the hammer (*malleus*), *m*; the anvil (*incus*), *i*; and the stirrup (*stapes*), *s*.

The handle of the hammer is attached to the tympanum and the base of the stirrup rests against the membrane of the *oval window*, a small oval opening into the inner ear. The middle ear communicates with the external air by means of the *Eustachian tube E*, which leads to the upper part of the throat.

The internal ear, or labyrinth, is seated deep in the skull, and communicates with the middle ear by two openings in its bony case, the *oval window*, and the *round window*, *fr*. It consists of three parts: the *vestibule*, *V*; the *semicircular canals*, *h*, *vp*, *va*; and the *cochlea*, or snail shell, *c*, all of which are filled with a watery fluid.

The cochlea is a tapering tube, coiled up like a snail shell, and divided longitudinally into two compartments. These are formed by a bony partition, extending out from the axis of the spiral, and two membranes joined to its edge and attached to the walls of the tube. In the fluid of the cochlea between the two membranes are found about 3000 rods of different length, known as the rods of Corti. These rods are fastened to the lower membrane, standing in two parallel rows, which extend its whole length. The rows lean toward each other, each rod being joined to its opposite neighbor. A nerve filament from the brain is connected to each pair of rods. A large number of stiff, elastic hairs are also found floating in the fluid of the vestibule and attached to the membrane on its sides.

The process of hearing begins in the transmission of the sound waves to the drum of the ear through the air. From the eardrum the vibratory motion is transferred by the small bones, to the yielding membrane of the oval window, and transmitted to the enclosed liquid of the vestibule and the cochlea, by which it is transmitted to the nerve filaments attached to the rods of Corti. It is thought that the function of these rods is to resolve sounds into their components and to report the individual components to the brain. It is assumed that such resolution is effected through resonance; each pair of rods responding to some specific tone. The hairlike extensions of nerve filaments in the vestibule are thought to aid in the perception of sounds that are lacking in tonal or musical quality.

The semicircular canals *h*, *va*, and *vp* (Fig. 78) are so placed that one lies in a horizontal plane, the second in a vertical plane from front to back, and the third in a vertical plane from right to left. These canals seem to have no connection with the sense of sound, but are regarded as the organ of the sense of equilibrium. Animals from which one or more of these tubes have been removed seem to experience great difficulty in maintaining their equilibrium.

HEAT

INTRODUCTION

CHAPTER XVII

NATURE OF HEAT

146. Nature of Heat. When a vessel containing cold water is placed over a fire, the water first becomes warm, then hot, and finally begins to boil, owing to the passage of heat from the fire to the water. After the water has begun to boil, it will not become appreciably hotter, no matter how long the boiling may be continued. If the fire be made more intense, the water will only boil more rapidly. There is no reason to suppose that heat ceases to pass into the water when the boiling begins. On the contrary, it seems clear that large quantities of heat are absorbed in order to keep the water boiling, and that the only effect which this heat produces is the continuous formation of steam from the boiling water.

If the experiment be begun with a vessel filled with pieces of ice, the ice melts, but if heat be added slowly and the contents of the vessel be continually stirred, the mixture of ice and water remains practically as cold as the ice, so long as there is any ice present. Only after the ice is all melted does the water begin to grow sensibly warmer.

From this we see that the addition of what we call heat to a body produces two distinct effects: (*a*) A change in the condition of the body affecting the sensation of warmth and cold. Such expressions as "cold," "cool," "warm," "tepid," and "hot," all refer primarily to this sensation, or they characterize the degree of hotness of the body. In scientific language the thermal condition of a body is designated as its *temperature*, and the hotter a body is, the higher its temperature is said to be.

(*b*) A change of state, such as the formation of water from ice or steam from water.

The addition of heat to a body produces many other effects. For example, if a thermometer be placed in hot water, the mercury rises in the tube, showing that the volume of the mercury increases on the addition of heat.

Again, if instead of the thermometer, we place in the hot water a thin glass flask completely filled with air or any other fluid, and closed by a stopper to prevent change of volume, we shall see that one of two things will happen: either the stopper will be driven out or the flask will be broken. This shows that the pressure of an enclosed mass of fluid increases on heating, provided the volume be kept constant.

In fact, almost every physical property of a body will change when the body is heated, but the examples cited are sufficient to show that change of temperature is only one of the many effects which heat can produce, and that heat and temperature are two entirely different physical quantities.

Heat was formerly supposed to be a subtle, imponderable fluid called "caloric," which upon entering a body produced the effects considered above (Black,[1] 1728–1799). Many of the terms used in the following pages date back to the caloric theory. At the close of the eighteenth century, however, a number of experiments by Count Rumford[2] (1753–1814) and Sir Humphry Davy[3] (1778–1829) proved definitely that *heat may be produced by work done against friction*, and that *the quantity of heat produced bears a constant ratio to the quantity of work expended in overcoming friction.*

These experiments disposed of the caloric theory of heat, once for all, and by connecting the production of heat directly with the known laws of mechanics, placed this branch of physics upon a sound scientific basis. At present *heat is known to be a form of energy which shows itself chiefly through the effects of temperature changes produced by it in different bodies.*

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[1] Black, *Lectures on the Elements of Chemistry*, vol. i, p. 156; 1803.

[2] Rumford, *Phil. Trans.* 1798 and 1799.

[3] Davy, *Complete Works*, vol. ii, p. 11.

147. Molecular Theory of Heat. The view that heat is a form of energy is closely connected with the molecular theory of the constitution of matter. In this theory it is assumed that the molecules of a body are in continuous and irregular motion. In a solid this molecular motion is restricted to a small space about the position of equilibrium of the molecule which, in general, may be considered as fixed in the body. In liquids there is more molecular freedom, so that the molecules are able to slip past each other and move about from point to point within the liquid, as is shown in the phenomena of diffusion (Art. 98). In a gas the molecules are assumed to have great freedom of motion and to be held together only by the application of some external force. In gases of ordinary density the mutual force of attraction between the individual molecules is conceived to be extremely small, and to exert little or no tendency to draw the mass of gas together, until the gas has been compressed by external force almost to the point of liquefaction.

It is also to be noted that the motion of the molecules is assumed to be not only *continuous* and *irregular*, but *blind* and *undirected* as well. By this we mean that the motion of the molecules of an enclosed gas is not comparable to that of persons in a crowd or of bees in a swarm, where the motion, while both continuous and irregular, is yet such that the individual avoids collision with his fellows. The molecules of the gas, however, are continually colliding with the walls of the enclosing vessel and also with each other.

The average distance traversed by a molecule between collisions is termed *the mean free path of the molecule.* Obviously this mean free path decreases as the density of the gas increases; when the gas has been compressed until liquefaction occurs the mean free path is greatly restricted, while in the case of a solid the molecule is closely hemmed in by its neighbors, and its motion becomes in general an irregular vibration about its position of equilibrium, in which it is held by the molecular attraction of its fellows. However, diffusion has been observed between solids, especially at higher temperatures, showing that the molecules of a solid have still a certain amount of freedom.

The energy of the molecule, like the energy of the whole mass, may take either the potential or the kinetic form. Similarly, the kinetic energy of the molecule may be either translational or rotational in character. If heat be added to a body, there results an increase in the molecular energy of the body at the point at which the heat enters the body. This increase of energy soon becomes distributed throughout the body, owing to the constant collisions of the molecules. If the average molecular kinetic energy per unit volume of the body has become the same in all parts of the body, the body is said to be uniformly heated. The rise in the temperature of a body is thus explained by an increase in the kinetic energy of the molecules.

If we strike a piece of iron with a heavy hammer, the mass kinetic energy of the hammer suddenly disappears and in its stead the molecules of the hammer and of the piece of iron are thrown into more violent agitation, or the hammer and the iron are heated. In this way we see that the mass kinetic energy of the hammer has been transformed into molecular kinetic energy in the hammer and in the iron.

Although the molecular theory of heat furnishes a satisfactory explanation for the phenomena of heat, yet the laws and relations derived in the following paragraphs are entirely independent of any theoretical explanation of the processes of heating and cooling.

TEMPERATURE

CHAPTER XVIII

THERMOMETRY

148. Temperature. After a body has either gained or lost heat, its thermal condition has undergone a certain change. Neglecting for the present any possible change of state, the most noticeable change in the body produced by this gain or loss in heat is a change in its temperature. It therefore becomes a matter of first importance to arrive at some reliable means for the measurement of this temperature change.

Our temperature sense furnishes us an approximate idea of the thermal condition of a body, although this estimate is greatly influenced by other factors. Thus on a cold day a piece of metal feels very cold to the touch, while a piece of wood seems less cold and a piece of wool may even appear warm. On a hot day these sensations would be exactly reversed, since the metal would then seem hot, the wood less so and the wool would appear cool. From this it is clear that our sensations of heat and cold depend not so much upon the actual temperature of the body in question, as upon the rate at which heat passes from the hand to the body, or from the body to the hand in either case. The apparent differences in temperature are due to the fact that this rate of transmission of heat from or to the hand in the foregoing examples is much greater in the case of metals than in the case of such substances as wood or wool.

A more reliable and accurate method of comparing temperatures is found in the measurement of some one of the many physical changes accompanying a change in the temperature of a body. *In this sense temperature is the physical measure, on an arbitrary scale, of that condition of a body, capable of affecting our sense of warmth and cold.* An instrument used for measuring temperature is called a thermometer, or in the case of very high temperatures, a pyrometer.

It is evident that a thermometer indicates directly, simply its own temperature, and only indirectly the temperature of the medium in which it is immersed. However, the *law of equalization of temperature* states that, when two bodies are in close thermal contact, and do not lose heat rapidly to their surroundings, nor gain heat from outside sources, any difference in temperature will soon disappear unless one of the bodies either gains or loses heat more rapidly than the other. Consequently a thermometer immersed in a fluid will soon assume the temperature of the fluid, and its readings, after becoming constant, may safely be assumed to indicate the temperature of the fluid in which it is immersed.

149. The Mercury-in-glass Thermometer. The most common form of thermometer is the mercury-in-glass thermometer, or a thermometer in which mercury is used as the thermometric substance, whose expansion in contrast to the expansion of the enclosing glass bulb is to be used to measure temperature changes. The *principle of temperature measurement* is, that changes in temperature, corresponding to equal changes in the apparent volume of mercury in glass, shall be called equal changes in temperature. As a thermometric substance, mercury has certain advantages over other liquids:

(*a*) It is easily obtained in a pure state.

(*b*) It does not stick to the glass.

(*c*) It is opaque and easily seen in a capillary glass tube.

(*d*) Its expansion with change of temperature is quite large.

(*e*) To each temperature there corresponds one and but one definite volume.

(*f*) It requires but a small amount of heat to raise its temperature through a given range, and so the introduction of the thermometer does not appreciably change the temperature of the substance whose temperature is to be measured.

In order to render small changes in volume noticeable, the mercury is enclosed in a glass bulb ending in a tube of capillary bore. The sensitiveness of the thermometer is greater, the larger the bulb and the finer the capillary tube.

150. Limitations of the Mercury-in-glass Thermometer. Mercury freezes at − 39°.C, and consequently cannot be used to measure temperatures below this point. Instead of mercury, alcohol, toluene or pentane are used for low-temperature measurements. Pentane remains liquid even at − 200° C. Since these liquids expand at rates very different from that of mercury, a careful calibration of such thermometers is necessary. Mercury boils at 356°.7 C, under atmospheric pressure, but the boiling may be prevented by an increase of pressure. In order, therefore, to use mercury thermometers for still higher temperatures, it is only necessary to fill the space above the mercury in the tube of the thermometer with some inert gas, usually nitrogen. Upon the expansion of the mercury the enclosed nitrogen is compressed and the mercury is prevented from boiling. Mercury-in-glass thermometers reading up to 500° C are not uncommon, the only limit being the strength of the glass envelope. Glass becomes soft at temperatures higher than 550° C, but by using tubes of fused quartz which softens at about 1100° C, thermometers reading up to 700° C have been made.

Another limitation of the mercury-in-glass thermometer is due to the nature of the glass envelope itself. Glass bulbs when blown, contract slowly for months and even years. This slow contraction changes the volume of mercury in the bulb, and so changes all readings of the thermometer by a certain amount. In this way a thermometer comes to read too high, and correction for this error must be made. This defect may be largely overcome by allowing the bulbs to lie several years before filling them.

A second defect of the glass bulb arises from sudden and large temperature variations. Thus if a thermometer be placed first in ice water, then in boiling water, and again in ice water, the two readings in the ice water will not agree. This is due to the fact that the glass bulb, when expanded by the boiling water, does not return at once to its original volume when again immersed in the ice water. Since all readings are now too low, this defect is termed the " depression of the zero point." It may be almost entirely eliminated by the use of a special glass for making the bulbs.

151. Other Forms of Thermometer. As we shall see later, many other forms of thermometer have been devised and used for accurate scientific investigation or for use under conditions which would render the mercury thermometer unsuitable. The most satisfactory standard for temperature measurement is the change in pressure of a definite volume of hydrogen gas, under known conditions, and this thermometer, known as the standard hydrogen thermometer, has been generally accepted among scientific men (Art. 165).

Of the various other forms of thermometer used for special purposes, a few will be mentioned here, but each will be more fully described in connection with the physical principle involved in its construction. Among these are the strain thermometer, depending upon the linear expansion of solids; the gas thermometer, based upon the increase in volume or the increase in pressure of a mass of gas when heated; the resistance thermometer, involving the increase of electrical resistance with increase of temperature; the thermo-couple, depending upon the principles of thermo-electricity; and finally the radiation thermometer, based upon the laws of radiation, by means of which it has been rendered possible to measure the temperature of a remote source of energy, as for example, the sun or the moon, whose radiations come to us over vast distances.

152. Centigrade and Fahrenheit Scales. After the thermometric substance and the principle of temperature measurement have been agreed upon, we are still free to select not only the value of a degree, or unit of change of temperature, but also the zero point from which to count.

It has been shown by experiment that the temperatures of pure melting ice, and of steam issuing from boiling water, are constant under a given pressure. These two temperatures, corresponding to a barometric pressure of 76 cm of mercury, are called the "freezing point" and the "boiling point." They are known as the fixed points of a thermometer, and the temperature difference between them is called the "fundamental interval."[1]

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[1] For experimental determination of the fixed points, see Manual, Exercise ~~32~~.
37

In all scientific work the *Centigrade scale* is used, which divides the fundamental interval into 100 equal parts or degrees, and takes the freezing point as the zero of the scale. All temperatures below this are written with the negative sign. The subdivision into 100 parts was first suggested by Celsius of Upsala, in 1742. For this reason, this scale is sometimes called the "Celsius scale," although he choose the boiling point as zero and the freezing point as 100.

In English-speaking countries a different scale, invented by Fahrenheit of Danzig (1686–1736), is used, and it is called the *Fahrenheit scale.* In this, the fundamental interval is divided into 180°, and the normal freezing point called 32°. It follows that the temperature of steam over water boiling under normal atmospheric pressure is 212° F.

The reading for a given temperature on one of these scales may easily be changed to the corresponding reading on the other scale. Call F the reading on the Fahrenheit, and C the reading on the Centigrade scale; then, since there are $F-32$ and C scale parts above or below the freezing point, we have (Fig. 79)

$$\frac{F-32}{C}=\frac{180}{100} \qquad (216)$$

or

$$F=\tfrac{9}{5}C+32 \text{ and}$$
$$C=\tfrac{5}{9}(F-32) \qquad (217)$$

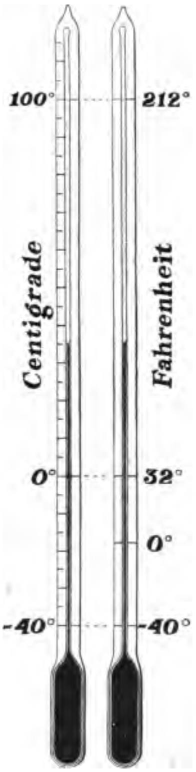

FIG. 79.

In a third scale, called the *Réaumur scale*, the fundamental interval is divided into 80 parts, with the freezing point as zero. This scale, however, is little used in this country.

153. Maximum and Mimimum Thermometers. Frequently it is important to know only the highest or lowest temperature reached during a certain time interval. In Six's thermometer (Fig. 80) the large bulb A is filled with glycerine, while the narrower tube is bent in a *U*-shape and partly filled with mercury. On the other end of the *U*-tube there is a small bulb ***B***, partially evacuated, and also containing some glycerine. In the glycerine above the mercury, on both sides of the *U*-tube, two thin, short iron wires are placed, each carrying a small spring which presses against the wall of the tube. With rising temperature, the mercury on the left-hand side pushes the wire above it upward, but on receding, as the temperature falls again, leaves it in the highest position reached. With decreasing temperature, the mercury on the right-hand side rises and pushes its wire up until the minimum temperature is reached. The lower ends of the wires on the left and right hand side, respectively, then give the maximum and minimum temperatures reached since the last setting. A new setting is made by pulling the wires down to the surface of the mercury, in each case by means of a magnet.

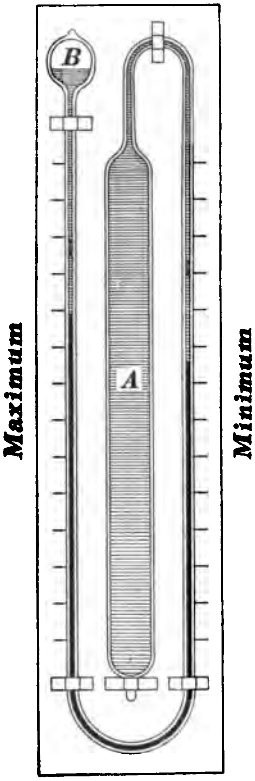

Fig. 80.

The United States Weather Service uses two separate thermometers. In the maximum thermometer, the bore of the tube is constricted near the bulb. With rising temperature, the mercury pushes through the constriction, but as soon as the temperature falls, the mercury breaks at this point, leaving the whole thread in the tube. Its reading gives directly the maximum temperature. The minimum thermometer is an alcohol thermometer, carrying within the alcohol a thin iron wire, which, by the surface tension of the alcohol, is pulled back to the lowest point reached. The alcohol on rising flows past the wire, leaving it at its lowest position. The end of the rod nearest the alcohol surface gives the minimum reading. Both thermometers are placed nearly horizontal (Fig. 81). The maximum thermometer is reset by whirling it around the pin at its upper end. The centrifugal force drives the mercury back into the bulb. To replace the index in the minimum thermometer next to the alcohol surface, the bulb of the thermometer is raised until the wire slides forward.

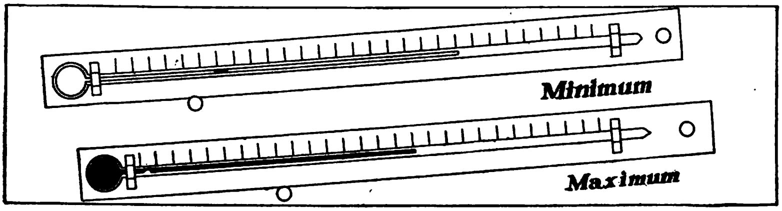

FIG. 81.

Clinical thermometers are also maximum reading thermometers, in which the mercury, broken by the constricted tube, gives the maximum temperature indicated by the instrument while in contact with the body of the patient. The mercury is brought back into the bulb by centrifugal force.

not necessarily. a jar
will answer

CHAPTER XIX

EXPANSION

154. Linear Expansion of Solids. In solids, changes in length due to a change in temperature are, in general, very small. This change, however, may be shown to a large audience by clamping one end of a metal bar (Fig. 82) about one meter long to a solid support and allowing the other end to rest upon a fine cambric needle, placed at right angles to the bar and free to roll upon a piece of plate glass. A long light pointer of paper attached to the needle will show its motion very clearly. On heating the bar, its free end will advance slightly, rolling the needle forward, and its motion, greatly magnified, is shown by the paper pointer. When allowed to cool the bar contracts, and the pointer moves in the opposite direction.

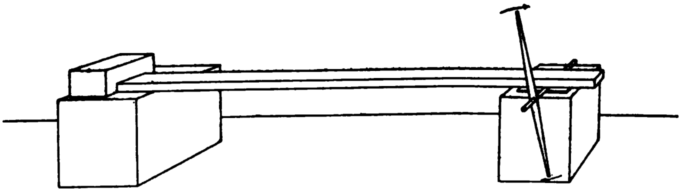

FIG. 82.

For the accurate determination of the linear expansion of solids, very refined methods are required. The best of these are optical methods involving interference of light, and the change in length caused by a change in temperature is expressed in terms of the wave length of the light used.[1]

If L_1 and L_2 represent the lengths of a solid bar, at temperatures t_1 and t_2, *then the change in length*, $L_2 - L_1$, is proportional to the original length L_1 and is nearly proportional to the change in temperature $t_2 - t_1$. This is expressed by the equation

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[1] *For a simple method for measuring linear expansion see Manual, Exercise* 43

$$L_2 - L_1 = \alpha L_1 (t_2 - t_1) \tag{218}$$

or

$$\alpha = \frac{L_2 - L_1}{L_1 (t_2 - t_1)} \tag{219}$$

The proportionality factor α is called *the coefficient of linear expansion*, and may be defined as the *change in length per unit length per degree*. The length at 0° C is usually taken as the original length, so that the length L_t, at any other temperature $t°$, is given by the equation Roman

$$L_t = L_0(1 + \alpha t) \tag{220}$$ Rom. ?

However, α is not quite constant, but in general increases with increase of temperature, so that at higher temperatures the expansion per degree is somewhat larger than at lower temperatures. /-/ In any determination of this physical quantity, therefore, it should always be stated between what limits the observations were made.

TABLE VI

MEAN COEFFICIENT OF LINEAR EXPANSION BETWEEN 0° AND 100° C PER DEGREE C

SUBSTANCE	$\alpha \times 10^6$	SUBSTANCE	$\alpha \times 10^6$
Copper	17.1	Glass	7 to 9
Iron	12.1	Jena thermometer glass	8.0
Platinum	9.3	Porcelain	3 to 4
Zinc	3.0	Fused quartz	0.56
Brass	18.4	Ice	52.0
Nickel-steel (36 % Ni)	1.0	Hard rubber	80.0

155. Practical Importance of Expansion. The forces exerted by bodies expanding or contracting under change of temperature are very great. Allowance must therefore be made for expansion in laying railroad rails or in designing girders for bridges. In a system of steam or hot water pipes expansion joints must be inserted. In riveting iron plates, the rivets are placed in the holes while red-hot and hammered into shape before they cool appreciably; on cooling, the rivets grip the plates together with terrific force. Carriage tires are put on the wheel while hot, and the glass stopper of a flask, when ‘stuck,” may often be loosened by gently heating the neck of the flask.

Glass vessels are easily broken when suddenly heated or cooled, because the brittle glass cannot support the internal strains produced, before the temperatures of the outer and inner portions have become equalized. Porcelain will stand sudden changes much better, and fused quartz, whose coefficient of linear expansion is only $\frac{1}{14}$ of that of glass, may even be heated red-hot and plunged immediately into cold water without being broken. Invar steel is an alloy of nickel and iron with very small temperature coefficient and is used extensively for the construction of pendulums, steel tapes, etc.

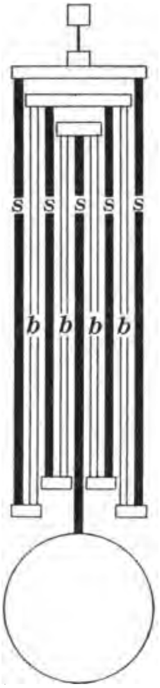

FIG. 83.

156. Further Applications. Any variation in the length of the pendulum will change the rate of a pendulum clock. Several devices have been employed to maintain the length constant with changing temperature. In the gridiron pendulum (Fig. 83) the lengthening of the rods, marked *s*, usually of steel, lowers the bob, while the expansion of the rods *b*, usually of brass, raises the bob. If the total effective lengths of the two systems of rods be called L_s and L_b, and α_s and α_b be their coefficients of linear expansion, the length of the pendulum remains the same with change of temperature, if the expansion of the brass be made equal to the expansion of the steel, or if,

$$\alpha_s L_s(t_2 - t_1) = \alpha_b L_b(t_2 - t_1) \qquad (221)$$

This condition is fulfilled, when

$$L_s/L_b = \alpha_b/\alpha_s \qquad (222)$$

In the mercury compensating pendulum (Fig. 84) the lengthening or shortening of the rod is counteracted by a rise or fall of the center of gravity due to the expansion of mercury, either in a vessel forming the bob of the pendulum or enclosed in the hollow stem of the pendulum rod.

In chronometers and the better grade of watches the rate is kept constant by making the rim of the balance wheel of two different metals, the one with the larger coefficient being on the outside (Fig. 85). If the temperature rise, the rim will bend so as to decrease the diameter and consequently the moment of inertia, producing a more rapid motion of the wheel. This is made to balance exactly the effect of temperature upon the elasticity of the spring and the effect of the expansion of the diameter, both of which would result in a slower motion of the wheel.

The same principle is employed in the so-called strain thermometers or "metallic" thermometers, frequently used in self-recording instruments. Their action may easily be understood from Fig. 86.

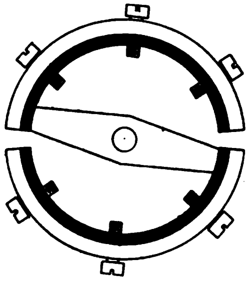

Fig. 85.

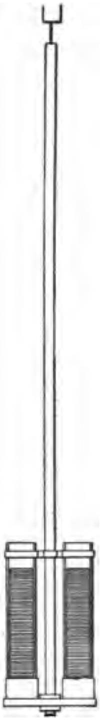

Fig. 84.

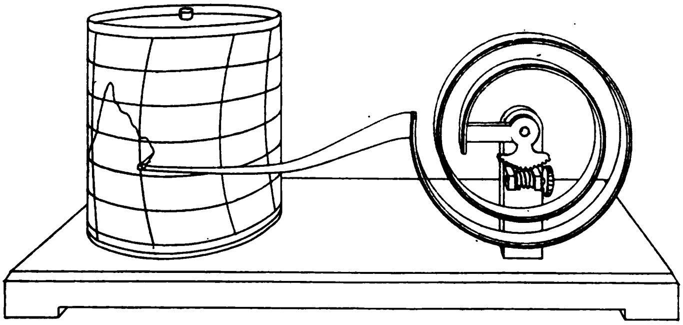

Fig. 86.

157. Cubical Expansion of Solids. An isotropic body is one whose physical properties are the same in all directions. Consequently an isotropic body, when heated, expands uniformly in all directions. Let us consider a parallelopiped cut from such a body. Let the three dimensions of this parallelopiped at 0° C, be L'_0, L''_0, and L'''_0; at t° C these dimensions will have become

$$\left.\begin{aligned} L'_t &= L'_0(1+\alpha t) \\ L''_t &= L''_0(1+\alpha t) \\ L'''_t &= L'''_0(1+\alpha t) \end{aligned}\right\} \tag{223}$$

and

$$L'_t \cdot L''_t \cdot L'''_t = L'_0 \cdot L''_0 \cdot L'''_0 (1+\alpha t)^3 \tag{224}$$

$$= L'_0 \cdot L''_0 \cdot L'''_0 (1+3\,\alpha t) \tag{225}$$

since we may neglect terms containing α^2 and α^3 in comparison with $3\,\alpha$.

Also since

$$V_0 = L'_0 \cdot L''_0 \cdot L'''_0 \tag{226}$$

and

$$V_t = L'_t \cdot L''_t \cdot L'''_t \tag{227}$$

we have

$$\left.\begin{aligned} V_t &= V_0(1+3\,\alpha t) \\ &= V_0(1+\beta t) \end{aligned}\right\} \tag{228}$$

where

$$\beta = 3\,\alpha. \tag{229}$$

The quantity β is called the *coefficient of cubical expansion.*

In cubical as in linear expansion there exists no strict proportionality between increase of volume and increase of temperature. This is the reason why the readings of thermometers filled with different liquids, such as mercury and alcohol, do not exactly agree. If an empty flask be heated, it expands as if it were solid throughout, and the internal cavity increases by the same amount as would a solid body of the same form as the cavity and made of the same material as the flask.

158. Anomalous Expansion. Most substances expand upon being heated. There are, however, some exceptions to the rule such as iodide of silver, between − 60° and + 142°, cuprous oxide, below 4°, diamond, below − 39°, and fused quartz, below − 80°. The most important exception is water (Art. 160).

A rubber tube, when stretched to twice its original length or more, contracts when steam is passed through it. Accurate measurements, however, show that its volume increases. The rubber has become *anisotropic* and its expansion in different directions is different, just as in all crystals, except those of the regular system.

159. Expansion of Liquids. The cubical expansion of liquids and gases is much larger than that of solids. If a vessel containing a liquid be heated, both liquid and vessel expand, and the rise of the liquid in the vessel, as for instance the rise of mercury in a thermometer, indicates merely the *relative* expansion of the two substances. The effect of the expansion of the vessel is easily shown by filling a flask completely with a colored liquid, and closing it by a stopper through which a narrow tube passes, so that the liquid stands at a certain height in the tube. If now the flask be placed in warm water, the liquid will be seen at first to sink in the tube and then to rise, owing to the fact that the flask is heated first. Owing to the expansion of the vessel, the rise of the liquid in the tube is less than it would be if the vessel did not expand. The real coefficient of cubical expansion of the liquid, β, is therefore the coefficient of the vessel, β_v, plus the apparent coefficient β_a of the liquid, or

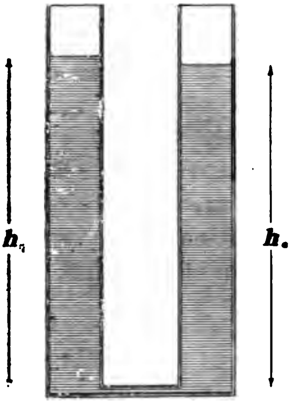

FIG. 87.

$$\beta = \beta_v + \beta_a \tag{230}$$

The real coefficient of expansion of the liquid may be determined directly by heating to different temperatures the two arms of a *U*-tube (Fig. 87) partly filled with the liquid in question. The pressures due to the columns on both sides balance in the horizontal tube. Let h_0 be the height of the meniscus above the horizontal tube at the lower temperature t_0, and let the density of the liquid on this side be d_0; let h_t, t, and d_t be the corresponding values on the other side. Then

$$h_0 d_0 g = h_t d_t g \tag{231}$$

But for a given mass M of the liquid

$$M = V_0 d_0 = V_t d_t \tag{232}$$

or

$$\frac{V_t}{V_0} = \frac{d_0}{d_t} = \frac{h_t}{h_0} \tag{233}$$

Now from (228) $$V_t = V_0[1 + \beta(t - t_0)] \tag{234}$$

Therefore, if t_0 be chosen as 0° C,

$$\frac{h_t}{h_0} = \frac{d_0}{d_t} = \frac{V_t}{V_0} = 1 + \beta t \tag{235}$$

and $$h_t = h_0(1 + \beta t) \tag{236}$$

from which β can easily be found. In this way the coefficient of expansion of mercury has been found to be 0.0001818 per degree.

The coefficient β increases considerably with increasing temperature and becomes quite large near the boiling point. Equation (234) should therefore be considered only as an approximation and, in the case of liquids, it is better to write,

$$V_t = V_0(1 + \beta' t + \beta'' t^2) \tag{237}$$

Thus for alcohol $\beta' = 1020 \times 10^{-6}$ and $\beta'' = 200 \times 10^{-8}$,
for ether $\beta' = 1480 \times 10^{-6}$ and $\beta'' = 350 \times 10^{-8}$.

160. Maximum Density of Water. Water under 4° C forms an important exception to the general rule that bodies expand upon being heated. When cooled, under atmospheric pressure, from higher temperatures, it contracts, reaches the smallest volume at 4° C, and expands again upon further cooling. Figure 88 shows the variation of its specific volume (Art. 5), with temperature. If freezing be prevented by avoiding mechanical disturbances (Art. 195), the expansion continues below 0° C, as is indicated by the dotted line.

When a large body of water, as a lake, is cooled, the cooler, upper layer sinks to the bottom until a temperature of 4° C is reached. After this the cold water remains at the surface and it is there that the formation of ice begins, while the water at greater depths is still at 4° C. The temperature of maximum density is considerably lowered by salts dissolved in the water. Thus sea water freezes before it reaches its maximum density. The temperature of the ocean at great depths is 2°.5 C.

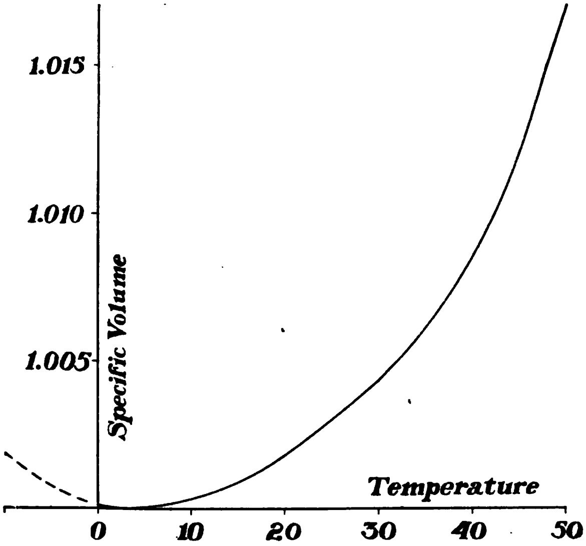

FIG. 88.

161. Expansion of Gases. Law of Gay-Lussac. In 1802 Gay-Lussac discovered that gas, when heated under constant pressure, expands equally for all equal differences of temperature, and that all gases possess the same coefficient of cubical expansion. This is known as the law of Gay-Lussac,[1] although it is frequently referred to as the law of Charles. The law is by no means exact. More recent investigations have shown that the coefficient of expansion is considerably larger in the case of gases which may be readily liquefied, such as chlorine and sulphur dioxide, than in the case of air, hydrogen or oxygen (Table 7). For the latter gases the coefficient of expansion has the nearly constant value 0.00367 per degree. This is considerably larger than the coefficient of expansion of solids or liquids under ordinary conditions. If we denote the coefficient of expansion of a gas under constant pressure by α_p, the general equation (235) may be written

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[1] *Ann. Chim. et Phys.*, 1st ser., vol. 43.

$$V_t = V_0(1 + \alpha_p t) \qquad (238)$$

and

$$d_t = \frac{d_0}{1 + \alpha_p t}. \qquad (239)$$

162. The Constant Pressure Gas Thermometer. Owing to its large coefficient of expansion, an enclosed mass of gas forms an extremely sensitive indicator of temperature changes, and so-called gas thermometers were used long before the appearance of mercury-in-glass thermometers. The air thermometer (Fig. 89) was probably invented by Galileo. In its simplest form it consists of a small bulb sealed to a long tube, which is mounted vertically with its lower end immersed in a suitable vessel filled with colored liquid. By expulsion of a few bubbles of air the liquid mounts into the tube. If the temperature of the bulb be varied slightly by placing the hand upon it, the colored index is promptly displaced through a considerable distance.

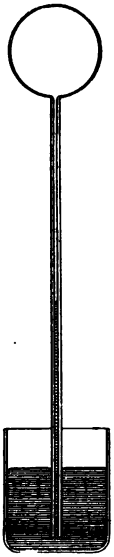

Fig. 89.

If we agree that changes in temperature, corresponding to equal changes in volume of a given mass of gas, under constant pressure, shall be called equal changes in temperature, we shall get a new temperature scale, but we should hardly expect this new scale to agree exactly with the one defined in Art. 149. The constant pressure gas thermometer is at present but little used, except as an indicator of temperature changes. The particular form known as the "Differential thermometer" (Fig. 90) may be made extremely sensitive, showing large displacements of the liquid index in the horizontal tube for slight differences of temperature between the two bulbs containing the gas.

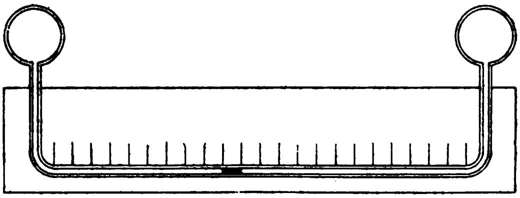

Fig. 90.

CHAPTER XX

INFLUENCE OF TEMPERATURE UPON THE PRESSURE OF A GAS

163. The Pressure Coefficient. When a mass of gas enclosed in a rigid vessel is heated, the increased kinetic energy of the gas produces an increased pressure in the vessel. When the volume is kept constant, the increase in the pressure exerted by the gas is very nearly proportional to the increase in temperature, as indicated by the mercury-in-glass thermometer,

or $$P_t = P_0(1 + \alpha_v t) \tag{240}$$

where α_v is called the pressure coefficient of the gas, and the subscript v denotes that the volume is to remain constant. For perfect gases, namely, those for which Boyle's law holds, the relation between the pressure coefficient and the coefficient of expansion is readily found. For such gases we have by equation (238)

$$V_t = V_0(1 + \alpha_p t)$$

Consider such a gas at 0° C, having a volume V_0 under a pressure P_0. Let the temperature be changed to $t°$, *keeping the pressure constant.* Then the product of pressure and volume at temperature $t°$ is

$$P_0 V_t = P_0 V_0\ (1 + \alpha_p t) \tag{241}$$

Take the same volume of gas at 0° C, and raise the temperature again to $t°$, but this time *keeping the volume constant.* Then the product of pressure and volume at the same temperature, $t°$, is

$$P_t V_0 = P_0 V_0\ (1 + \alpha_v t) \tag{242}$$

In both cases the gas is finally at $t°$, and according to Boyle's

law $$P_0 V_t = P_t V_0 = PV \tag{243}$$

This in connection with equations (241) and (242) shows that

$$\alpha_p = \alpha_v = \alpha \tag{244}$$

Hence for perfect gases the two coefficients are equal and we may write:

$$PV = P_0 V_0 (1 + \alpha t) \tag{245}$$

which is known as the Boyle Gay-Lussac law.

The following table shows that hydrogen, oxygen, nitrogen and air may under ordinary conditions be considered as very nearly perfect gases. Others, which may be readily liquefied, may be treated as perfect gases only at temperatures considerably above their points of liquefaction.

TABLE VII

Coefficients of Expansion and Pressure for Gases referred to Volume and Pressure at 0° C

P_0 ?

mercury ?

Substance	α_p $P = 76$ cm Hg	α_v $P_0 = 76$ cm Hg
Hydrogen	0.003660 per degree	0.003663 per degree
Oxygen	3674 " "	3665 " "
Nitrogen	3671 " "	3668 " "
Air	3671 " "	3665 " "
Carbon dioxide	3710 " "	3690 " "
Sulphur dioxide	3903 " "	3670 " "

164. The Constant Volume Gas Thermometer. If the volume of a given mass of gas be kept constant, equal to V_0, equation (245) becomes

$$P_t = P_0 (1 + \alpha t) \tag{246}$$

which, as previously shown, equation (240), is very nearly true for temperatures measured by the mercury-in-glass thermometer. This equation may be used as the basis of a new temperature scale. In order to do so we must make two fundamental assumptions.

(*a*) *Differences in temperature are to be taken as strictly pro-*

pressure of

portional to those differences in pressure which they produce in a mass of gas, maintained at constant volume.

(*b*) *The pressure* P_0 *is the pressure, exerted by this same mass of gas at the temperature, to be chosen as the zero of the new scale.* This will make the differences in temperature inversely proportional to the original pressure P_0, equation (249). If now we make our temperature readings to depend upon these assumptions, we see at once that equation (246) is made rigorously true. We should naturally expect any indicated temperature θ as read from this scale to be somewhat different from that given by the mercury-in-glass thermometer, which we denote by t. Then for a given temperature range t, read off on the mercury-in-glass thermometer, we have, substituting θ for t,

$$P = P_0(1 + \alpha\theta) \tag{247}$$

$$P - P_0 = \alpha P_0 \theta \tag{248}$$

and, finally,

$$\theta = \frac{1}{\alpha} \cdot \frac{P - P_0}{P_0} \tag{249}$$

If, now, we call as before the freezing point and boiling point of water 0° and 100°, and denote the pressure at the latter temperature by P_{100}, we have

$$P_{100} - P_0 = \alpha P_0 100 \tag{250}$$

and

$$\theta = 100 \frac{P - P_0}{P_{100} - P_0} \tag{251}$$

To find the temperature θ, therefore, it is only necessary to measure the pressures at the fundamental points and at the temperature to be determined.

In its simplest form (Fig. 91) this type of gas thermometer consists of a bulb, A, containing the gas, connected by a capillary tube to a U-tube filled with mercury. In order to insure constancy of volume of the enclosed gas, the mercury is adjusted for each reading, so that it just touches a pointer or mark, p, placed near the opening of the capillary tube, or on the tube itself. The pressures are calculated from the difference in height of the mercury in the two arms of the *U*-tube in each case. To the temperature θ, as computed from equation (251), there must be applied certain corrections, due to the expansion of the bulb, changes in barometric pressure, etc.[1]

165. The Standard Hydrogen Thermometer. Owing to certain serious limitations (Art. 150), the mercury-in-glass thermometer is not well adapted to serve as a final standard for measurements of temperature. In its stead, the constant volume gas thermometer has, by international agreement, been adopted as the standard instrument for temperature measurements. Further, since hydrogen fulfills most nearly the requirements for a perfect gas, it has been adopted as the thermometric substance, with the provision that the gas at freezing point shall be under a pressure of 100 cm of mercury.

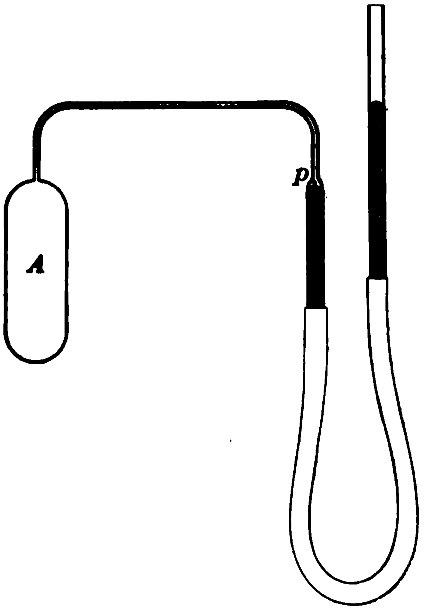

FIG. 91.

Temperatures measured by this thermometer are the final standards in all scientific work, and temperatures measured by any other means are usually "reduced to the hydrogen scale" when any high degree of precision is required. It should be said, however, that in the case of mercury-in-glass thermometers from the best makers, the readings do not differ greatly from those of the hydrogen scale, while the greater convenience of the mercury instruments more than compensates for its limitations, except in cases where extreme accuracy is required.

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[1] *See Manual, Exercise 40.*

pressure of

Throughout the remainder of this text, distinctions between temperatures based upon the mercury or the hydrogen scale will be disregarded. It is also to be understood that, in general, temperatures will be expressed in Centigrade degrees, unless it be expressly stated to the contrary.

166. The Zero of the Gas Scale. Absolute Temperatures. The selection of the freezing point of water as the zero point was an entirely arbitrary one. It is now possible to find a more rational zero point. From the principles developed in the discussion of the constant volume thermometer (Art. 164) it is clear that a decrease in temperature is indicated by a corresponding decrease in gas pressure. The rational zero for such a temperature scale would therefore be that temperature for which the pressure of the gas in the hydrogen thermometer becomes zero, assuming that hydrogen could remain a gas at that temperature. This zero point may readily be found from the equation

$$P_t = 0 = P_0(1 + \alpha t) \qquad (252)$$

Whence $$t = -\frac{1}{\alpha} = -\frac{1}{0.00366} = -273^\circ \text{ nearly.}$$

Thus we see that this zero is about 273° below the zero of the Centigrade scale. This point is called the zero point of the hydrogen scale, or the *absolute zero.* Temperatures measured on this scale are called *absolute temperatures.* Such readings are usually denoted by T, and are related to readings t, on the Centigrade scale through the equation

$$T = t + 273^\circ \qquad (253)$$

The terms "absolute zero" and "absolute temperatures" are not well chosen, inasmuch as this zero and all temperatures measured from it depend upon the use of some arbitrary thermometric substance.

In advanced physics the student will become acquainted with a temperature scale, called the thermodynamic scale, in which the temperatures are independent of any particular thermometric substance. T expressed in this scale, are "absolute" temperatures. They agree very closely with the absolute temperatures derived from the gas scale, and it seems unnecessary to distinguish between the two in this text. For a discussion of the thermodynamic scale the student is referred to more advanced textbooks.

In accordance with the molecular theory of heat, the pressure of an inclosed mass of gas is due to the bombardment of the sides of the vessel by the flying molecules. From this it follows that the pressure decreases as the molecular motion decreases, and consequently (Art. 147), at the absolute zero all molecular motion would cease. Hydrogen is not a perfect gas. It is due to this fact that the two sets of "absolute" temperatures referred to above do not exactly coincide.

167. The Gas Law. For a perfect gas at temperature t° we have found

$$PV = P_0 V_0 (1 + \alpha t) \qquad (254)$$

Introducing the absolute temperature, we have

$$1 + \alpha t = \alpha\left(\frac{1}{\alpha} + t\right) = \alpha(273 + t) = \alpha T \qquad (255)$$

$$PV = \alpha P_0 V_0 T = RT \qquad (256)$$

where R is a constant for a given mass of gas and may be called the *gas constant*.

For any other absolute temperature T',

$$P'V' = RT' \qquad (257)$$

By division of the last two equations we obtain the important relation

$$\frac{P\,V}{P'V'} = \frac{T}{T'} \qquad (258)$$

This is the mathematical statement of what is called the *gas law*. In words, it says that "*For a perfect gas the product of pressure and volume is proportional to the absolute temperature.*"

From equation (258) follow directly the two corollaries: "The pressure of a gas of constant volume varies directly as the absolute temperature" and "The volume of a gas under constant pressure varies directly as the absolute temperature."

Since ordinary gases at temperatures and pressures remote from their points of liquefaction behave nearly as perfect gases, these laws may be applied to such gases without introducing any serious error.

Problems

1. Reduce the following readings on a Fahrenheit thermometer to the corresponding readings on a Centigrade thermometer: + 2000°, + 104°, + 5°, − 49°.

2. Reduce the following readings on a Centigrade thermometer to the corresponding readings on the Fahrenheit thermometer: + 2000°, + 445°, − 10°, − 273°.

3. At what temperature do Fahrenheit and Centigrade thermometers read the same?

4. What are the coefficients of linear expansion of iron and copper when expressed in English units?

5. A certain railroad track is laid when the temperature is 32° F. If the rails are 30 ft long, how much space in inches must be left between them in order that they may just touch when the temperature is 122° F?

6. The steel cable from which the Brooklyn bridge hangs is more than a mile long. How many feet does a mile of its length vary between a winter day when the temperature is − 20° C and a summer day when it is 30° C?

7. In a gridiron pendulum the distance from the center suspension to the center oscillation is 99.3 cm. Supposing the whole pendulum except the vertical brass rods to be made of iron, how long must the brass rods be, for perfect compensation, all brass rods being the same length?

8. A clock with a seconds pendulum of iron is correct at 25° C. How many seconds per day will it gain if the temperature is 0°? What will be the result if the pendulum be made of nickel-steel?

9. How large a relative error is made if the moment of inertia of a brass cylinder, calculated from measurements at 10°, should be used in experiments carried on at 50°?

10. A certain glass measuring flask made of Jena glass has a capacity of one liter at 15°. What is its capacity at 40°?

11. The density of mercury at 0° is 13.596 g/cm^3. What is its specific volume at 35°?

12. Reduce to 0° a barometer reading of 74.5 cm taken at 19°, correcting both for the expansion of the mercury and for that of the brass scale.

13. A piece of brass weighs 17 g in air and 15 g in water at 10°. How much will it weigh in water at 30°?

14. A liter flask of Jena thermometer glass is calibrated at 0°. The cylindrical neck has a diameter of 1.5 cm. How far above the liter mark will the meniscus be, if the flask be filled with one kilogram of water at 30°? Spec. vol. of water at 0° C = 1.00012, at 30° C = 1.00435 cm^3 per gram.

15. A quantity of air, at atmospheric pressure and 0°, occupies 1000 cm^3. It is heated to 100°, under constant pressure. Find the increase in volume and the work done during the expansion.

16. What fractional part of the air in a room passes out when the air is heated from $-10°$ to 20°?

17. The density of air under standard conditions is 0.00129 g/cm^3. What will be the mass of air in a room whose dimensions are $8 \times 5 \times 3$ m, the temperature being 20° and the barometric reading 730 mm?

18. If the atmospheric pressure on a mountain top (Mont Blanc) be 380 mm of mercury, and the temperature $-10°$ C, what fraction of the normal quantity of air is contained in any given volume?

19. Suppose 20 cc of hydrogen gas to have been collected in a closed tube over mercury. The barometer reading corrected for temperature is 74.5 cm and the temperature of the gas is 24°. The mercury in the tube stands 12 cm above the mercury in the dish in which the tube is inverted. Reduce the volume of the gas to standard conditions.

20. A certain boiler boils dry, containing steam at 80 pounds per square inch, at 155° C, which in the absence of water may be considered as a perfect gas. If the temperature should rise to 600° C, would the boiler burst, if it can just support a pressure of 300 pounds per square inch?

QUANTITY OF HEAT

CHAPTER XXI

CALORIMETRY

168. The Unit of Heat. The Calorie. In calorimetry we are concerned with the measurement of definite quantities of heat. The idea of quantity of heat involves first the mass of the body to be heated. We know that the temperature of boiling water is the same, whether the vessel contain one liter or ten liters, and yet it will take a flame ten times as long to heat ten liters of water from zero to the boiling point, as to do the same for one liter, or (Art. 146), the heat required is ten times as great in the first case as in the second. Hence the quantity of heat needed to raise the temperature of a body through a given temperature interval is proportional to the mass of the body.

Again, we assume that the heat absorbed by a body when heated to a certain temperature is all given out when the body cools to its original temperature. Consequently when two kilos of water at 20° are mixed with two kilos of water at 0°, we expect the mixture to assume a final temperature of 10°, and experiment shows this to be true (Art. 173). If, on the other hand, we should substitute two kilos of iron or two kilos of mercury for the two kilos of water at 20° in the experiment, we should find that equal masses of different substances give out very different quantities of heat when cooled through the same temperature range. Thus a mass of mercury absorbs or gives out only *one thirtieth* as much heat as an equal mass of water would do when carried through the same temperature interval.

It is clear then, that the idea of the quantity of heat needed to heat a body involves three distinct things: (*a*) the mass of the body, (*b*) the material of which the body is composed, and (*c*) the temperature range through which it is to be heated. It is therefore necessary to specify all these things in defining the unit of heat. The accepted unit of heat is defined as *that quantity of heat which will raise the temperature of one gram of water from 15 to 16 degrees Centigrade. This unit is called a calorie.*

In the English system, the British thermal unit (B. T. U.) is defined as the quantity of heat needed to raise the temperature of one pound of water one degree Fahrenheit.

169. Thermal Capacity of a Body. It may be proven experimentally (Art. 173) that, unless changes of state occur, the quantity of heat added to a body is nearly proportional to the change of temperature produced. Thus if a given *body* be heated from $t_1°$ to $t_2°$, then the heat needed for this purpose is

$$H = C(t_2 - t_1). \tag{259}$$

The proportionality factor,

$$C = \frac{H}{t_2 - t_1} \tag{260}$$

is called the *thermal capacity of the body* and its unit is one calorie per degree.

170. Thermal Capacity of a Substance. It has likewise been found that the heat needed to produce a definite temperature change $(t_2 - t_1)$, in different masses of the same substance is proportional to the mass, or

$$H = cM\,(t_2 - t_1) \tag{261}$$

The proportionality factor

$$c = \frac{C}{M}, \tag{262}$$

which is evidently characteristic of the substance, is called the *thermal capacity of the substance*, and is *numerically* equal to the heat necessary to change the temperature of *one gram of the substance* one degree. Its unit is one calorie per gram-degree.

171. Thermal Capacity of Water. The above formulae are not quite exact, since in general the thermal capacity of a substance increases with the temperature. Since water has been chosen as the standard substance, we have numerically

$$\left.\begin{aligned} c_w &= 1 \\ M &= 1 \\ t_2 - t_1 &= 1 \end{aligned}\right\} \qquad (263)$$

Consequently the change of the thermal capacity of water is of especial interest. In Fig. 92 we have the thermal capacity of water plotted as a function of the temperature.

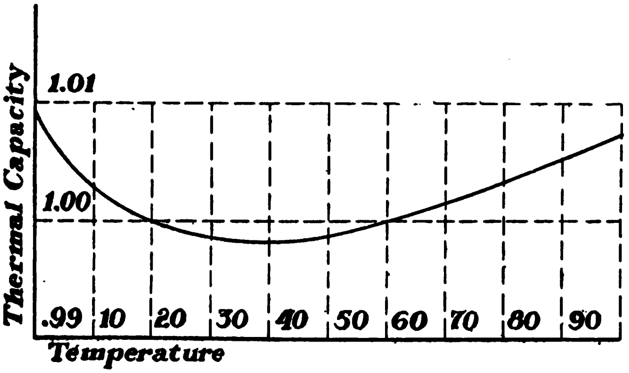

FIG. 92.

Though these variations are small, yet it is evident that a definite temperature must be specified in the definition of the calorie. The interval from 15° to 16° has been chosen since the calorie based upon this interval is the one one-hundredth part of the heat needed to raise the temperature of one gram of water from 0° to 100°. In this text it will be assumed that the thermal capacity of water c_w may always be taken as unity.

172. Specific Heat of a Substance. The specific heat s of a substance is *the ratio of the thermal capacity of the substance to that of water*, or

$$s = \frac{c}{c_w} \tag{264}$$

Since c_w is unity, the specific heat of a substance is numerically equal to its thermal capacity. In a loose sense the thermal capacity is often called specific heat, but the student should observe that the same relation holds between these two quantities as between density and specific gravity. The former is a definite physical quantity; the latter is simply a ratio or a pure number.

173. The Method of Mixtures. All heat measurements are based upon the following fundamental principle: *When two or more bodies, originally at different temperatures, are placed in thermal contact, and exchange of heat takes place exclusively between these bodies, the heat lost by one part of the system is equal to the heat gained by the other.* This is called the principle of equal heat exchanges.

Thus if a certain mass M of a substance, whose thermal capacity is c_1, be heated to a temperature $t_1°$ and then dropped into a mass of water M_w, at temperature t_2, where $t_2 < t_1$, an exchange of heat takes place and an intermediate temperature, t, is established. The body loses an amount of heat equal to $c_1M_1(t_1 - t)$, and the water gains an amount equal to $c_wM_w (t - t_2)$. Consequently we have

$$c_1M_1(t_1 - t) = c_wM_w(t - t_2) \tag{265}$$

and the specific heat of the substance is

$$s = \frac{c_1}{c_w} = \frac{M_w(t - t_2)}{M_1(t_1 - t)} \tag{266}$$

The water is contained in a vessel, supplied with a stirrer and a thermometer. These form the *calorimeter*. Their temperature is changed as well as that of the water. The heat given to them is equal to the sum of such terms as $cM(t - t_2)$, calculated for all parts of the calorimeter. The sum of the products of the specific heats into the mass, $\Sigma\frac{c}{c_w}M$ is called *the water equivalent* of the calorimeter, and must be added to the mass of water, M_w, in the experiment just described. Taking into account the heat effect of the calorimeter, we obtain then the general equation:

$$s = \frac{c_1}{c_w} = \frac{\left(M_w + \Sigma\frac{c}{c_w}M\right)(t - t_2)}{M_1(t_1 - t)} \tag{267}$$

TABLE VIII

Specific Heats of Solids and Liquids

Substance	s	Atomic W.	s × At. W.
Copper	0.094	63.6	5.98
Graphite	0.199	12.0	2.39
Iron	0.116	55.9	6.48
Lead	0.031	206.9	6.42
Mercury	0.033	200.0	6.60
Platinum	0.032	194.8	6.23
Tin	0.055	119.0	6.55
Zinc	0.094	65.4	6.15
Glass	0.200	——	——
Ice	0.505	——	——
Alcohol	0.602	——	——
Ether	0.547	——	——

174. Law of Dulong and Petit. The above table shows that the specific heats of different substances vary considerably. In 1819 Dulong and Petit announced the following law:[1] "The product of the specific heat of a substance into its atomic weight is the same for all elementary solid substances." The product thus obtained is about 6.

This law is by no means exact. Carbon, boron and silicon are exceptions. Since the specific heats vary with the temperature, these products will vary according to the temperature chosen.

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[1] *Ann. Chim. et Phys.*, 1819

Nevertheless this law suggests the possibility that the thermal capacities of atoms of different substances may be nearly the same.

For a more complete discussion of this law and others relating to the molecular heats of chemical compounds the student is referred to text books on physical chemistry.

175. Specific Heats of Gases. Very different values may be obtained for the specific heat of a gas, according to the conditions under which the gas is heated. If a gas be *allowed to expand* while it is being heated, it will do work against external pressure by virtue of that expansion. The energy needed to do this work must be supplied from the gas itself in the form of heat. Consequently the gas heated under this condition absorbs heat for two reasons: (*a*) *to produce change* of temperature, (*b*) *to furnish energy* to do the work of expansion.

On the other hand if the gas be heated and its *volume be kept* constant, no heat is absorbed except that needed to produce the rise in temperature. The two specific heats obtained under these two conditions are designated s_p, *the specific heat under constant pressure*, and s_v, *the specific heat under constant volume.* Of these two values, s_p is obviously the greater, by the amount of heat needed to produce the expansion. The ratio between these two values, for atmospheric pressure, usually denoted by γ, is of considerable importance in the computation of the velocity of sound in a gas (Art. 113).

TABLE IX

SPECIFIC HEATS OF GASES

SUBSTANCE	s_p	s_v	γ
Air	0.237	0.167	1.41
Hydrogen	3.410	2.418	1.41
Oxygen	0.217	0.147	1.41
Water vapor	0.421	——	1.30
Carbon dioxide	0.203	——	1.30
Alcohol	0.453	——	1.13

CHAPTER XXII

THE MECHANICAL THEORY OF HEAT

176. The Experiments of Joule and Rowland. We have seen (Art. 146) that heat must be considered as a form of energy and that heat is produced whenever mechanical energy is absorbed in overcoming friction. The first accurate experiments showing a definite quantitative relation between heat and mechanical work were made by Joule[1] (1818–1889). In his apparatus (1845) descending weights, *M*, were made to rotate a paddle wheel, *P*, immersed in a stationary calorimeter, *C* (Fig. 93). The resistance offered to the motion of the paddle was greatly increased by means of stationary vanes extending into the interior of the calorimeter. The work done by the total mass 2 *M*, descending through a height *h*, is 2 *Mgh*. The heat produced, measured in heat units, is

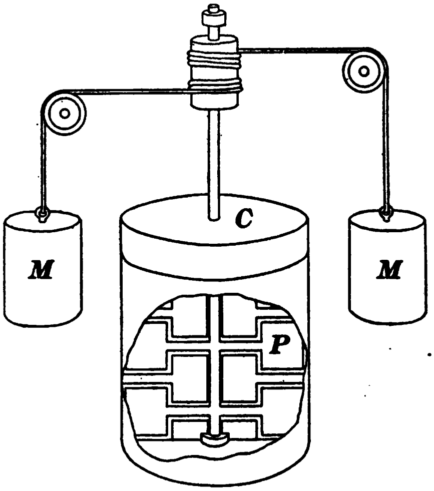

FIG. 93.

$$H = (t_2 - t_1)\Sigma cm \tag{268}$$

where *c* and *m* denote the thermal capacities and the masses of the different parts of the calorimeter and $t_2 - t_1$ the corresponding rise of temperature. Joule showed that for every unit of mechanical energy which disappears, the same quantity of heat is always produced.

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[1] *Phil. Trans. Roy. Soc.*, 1850.

In later experiments[1] (1878) he used a calorimeter which was free to turn about a vertical axis. If the paddle be turned, the friction in the water tends to rotate the calorimeter. The vessel, however, is kept stationary by means of two thin silk strings wound in a groove around the cylindrical vessel and leaving it in a tangential direction at two opposite ends of a diameter. These strings then pass over two light pulleys and carry at their lower ends weights M, which are adjusted until the calorimeter remains stationary, when the paddles revolve at a constant rate. If r be the radius of the groove, the moment of the mechanical forces counteracting the effect of the paddle wheel is $2\,Mgr$. The total work done in t seconds when the paddles are rotated n times per second against the torque due to the weights, is

$$W = torque \times angle = 2\,Mgr \times 2\,\pi nt = 4\,\pi ntrMg. \qquad (269)$$

In 1879 Rowland (1848–1901) made a series of classical experiments[2] with an improved form of Joule's later apparatus, obtained much more accurate results and also discovered the variation of the thermal capacity of water with change of temperature (Art. 171).

177. The Mechanical Equivalent of Heat. Joule's and Rowland's experiments, as well as many others of more recent date, have shown that *in any transformation of heat into work or of work into heat, the ratio between the numerical values of these two forms of energy is always a constant.* If the number of heat units be denoted by H, and the number of units of work by W, then

$$W \text{ mechanical units} = JH \text{ heat units.} \qquad (270)$$

If we combine equations (268), (269) and (270), we have

$$W = 4\,\pi ntrMg = J(t_2 - t_1)\,\Sigma cm \qquad (271)$$

or

$$J = \frac{4\,\pi ntrMg}{(t_2 - t_1)\,\Sigma cm}\,\frac{ergs}{cal} \qquad (272)$$

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[1] *Phil. Trans. Roy. Soc.*, 1878.

[2] *Proc. Am. Ac. Arts and Sci.*, 1879.

The value of J depends upon the units chosen.

Thus 778 foot pounds of work are equivalent to one B. T. U., or 0.427 kilogram-meter to one calorie.

From a critical study of these experiments Barnes concluded that

$$\text{One calorie} = 4.186 \times 10^7 \text{ ergs} = 4.186 \text{ joules}$$

The mechanical work corresponding to one heat unit is called the *mechanical equivalent of heat.*

We have thus obtained a new measure of heat in terms of mechanical units, and a new measure of mechanical work in terms of heat units. Thus

$$H \text{ calories} = JH \text{ joules} = 4.186\, H \text{ joules}$$

and

$$W \text{ joules} = W/J \text{ calories} = 0.239\, W \text{ calories}$$

178. The First Law of Thermodynamics. The experimental results given in the preceding paragraphs may be summarized thus: *When work is transformed into heat, or heat into work, the amount of work is always equivalent to the quantity of heat.* This is known as the first law of thermodynamics.[1]

Heat added to a body is considered as positive, heat given out by a body as negative; work done upon a body is positive; work done by the body, negative. Taking into account the signs, the mathematical expression for this law is:

$$W + H = 0 \tag{273}$$

when W and H are measured in the same units, or

$$W + JH = 0 \tag{274}$$

when W is expressed in mechanical, and H in heat units.

179. Equivalence of Energy and the Principle of Conservation. Having established the equivalence of mechanical work and heat, it is of the highest importance that we should grasp the full significance of this equivalence of energy which shows itself in every branch of Physics. Physical phenomena of every form depend upon the transference or the transformation of energy. We are now prepared to say that energy in any of its manifold forms may be reduced to an equivalent amount of heat and hence to an equivalent amount of energy of any other form. Thus an electric current of strength I, flowing for t seconds through a resistance R, produces a quantity of heat H, which is always proportional to I^2Rt. This quantity, measured in electrical units, is called electrical energy. In short if, in any physical phenomenon, energy of any form disappear, energy of some other form will always appear, and the energy of the new form is always equivalent to the energy of the old. No energy is lost.

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[1] This law was first clearly stated by Mayer in Liebig's *Annalen*, 1842.

This most important principle, first announced by Robert Mayer in 1845, is known as the principle of conservation of energy. It is this principle, verified by countless experiments, which underlies all physical phenomena, and which constitutes one of the grandest generalizations of modern science. Keeping in mind that energy added to a body is positive, and energy taken away is negative, the principle may be stated in these words: If in a system of bodies no reaction be allowed between the system and outside bodies, then the total amount of energy of the system is not changed by any reaction or transformation between the parts of the system.

The actual amount of energy in a body is not known, but for any change from a state A to a state B, the energy involved can be accurately determined and is the same regardless of the manner in which the change has been accomplished. For, let us assume that a system of two bodies be changed from one state to another during which change the first body gives to the second a certain amount of energy, and, that the system can be brought back to its original state by a method in which the second body returns to the first a smaller amount of energy than it received from it. The result of the complete cycle of changes would be an increase of energy possessed by the second body without an equivalent compensation on the part of the first, and this contradicts the first law of thermodynamics.

By a bold generalization, which admits of no proof by actual experiment, the principle is sometimes stated by saying that the energy of the universe is constant, or, that energy can neither be created nor destroyed.

180. Compression and Rarefaction of a Gas. Let a cylinder, closed by a piston, contain a given amount of gas. If the piston be suddenly pushed in through a *small distance* s, with a force F, the work done upon the gas is (Art. 35)

$$W = Fs = Pv \tag{275}$$

where P is the pressure and v is the change of volume. This mechanical energy expended upon the gas increases its energy and consequently the gas is heated.

On the other hand, if the gas be allowed to do work against a pressure P, increasing its volume by v, a quantity of heat equivalent to the work Pv is abstracted from the gas and it cools.

181. Free Expansion of a Gas. Joule connected two receivers, A and B (Fig. 94), by a tube, containing a stopcock, s. One of the vessels was exhausted while the other contained a gas under pressure. Both receivers were then immersed in a water calorimeter. Upon opening the stopcock the gas rushed into the vacuum and now filled both cylinders, but the calorimeter showed no change of temperature. From this Joule concluded that the energy of a given mass of gas is independent of the volume occupied.[1]

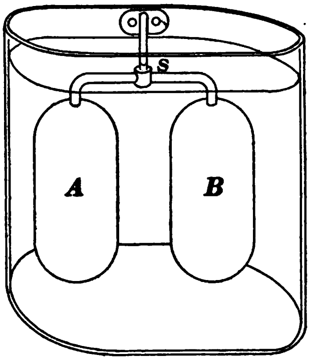

FIG. 94.

Placing each receiver in a separate calorimeter and repeating the experiment, the water in the vessel originally containing the compressed gas was cooled, and that in the other one was heated. Obviously the loss of energy on the one side equals the gain on the other.

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[1] This experiment was originally due to Gay-Lussac (*Mem. d'Arcueil*, 1807).

The results obtained by Joule in this experiment hold rigorously only for perfect gases. Ordinary gases do show a small change of energy upon expansion, but this change must be measured by more sensitive methods (Art. 186).

182. Isothermal and Adiabatic Expansion. If a gas expand very slowly while kept in close thermal contact with a heat reservoir of temperature $t°$, heat will constantly flow from the reservoir to the gas and keep it at the temperature $t°$ during the expansion. Such a process is called an *isothermal expansion* and is represented by line I (Fig. 95) in which the volumes of the gas are plotted as abscissae, and the corresponding pressures as ordinates. The equation of this line (Art. 167) is

$$PV = RT$$

The work done by the gas during a very small increase in volume v, equal to ab, is Pv, and is represented at the point Q of the curve in the figure, by the shaded area $Qcba$. Imagine a large number of such narrow strips drawn, side by side, and extending from the axis of volumes to the isothermal line. It is then clear that the work done by the gas during the isothermal expansion from P_0V_0 at point A to P_1V_1 at point B is equal to the sum of all the strips, and is therefore represented by the area $ABNL$ included between the isothermal line, the axis of volumes, and the two ordinates representing P_1 and P_0.

If the gas, on the other hand, be enclosed in a vessel whose walls prevent any flow of heat through them, the gas can receive no heat from the reservoir and will, therefore, cool during expansion. The pressure P_2 or ND, corresponding to volume V_1 will be smaller than P_1 or NB, and the line representing this change in the gas will *be steeper than the isothermal line.* Rhom ? Such an expansion is called *an adiabatic expansion*, and is shown by the adiabatic line II (Fig 95).

By the use of calculus it may be shown that the equation of an adiabatic line for a perfect gas is

$$PV^{\gamma} = \text{constant} \qquad (276)$$

where γ is the ratio of the two specific heats of the gas (Art. 175).

183. Evaluation of $C_p - C_v$. If a quantity of gas of mass M be heated under constant volume through a *temperature interval* of $t°$, the heat needed is Mc_vt calories. If it be heated under constant pressure, the heat needed is Mc_pt calories. According to Art. 181 the energy of a gas is independent of its volume and consequently the quantity of heat needed for the mere heating of the gas must be the same in both cases. But, when heated under constant pressure, the gas does work equal to Pv, and this energy must also be supplied by the heat added to the gas, hence

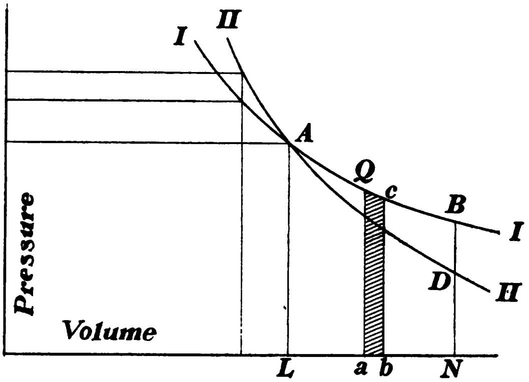

FIG. 95.

$$Mc_pt - Mc_vt = (c_p - c_v)\,Mt = Pv. \qquad (277)$$

According to Art. 163, equation 245, we have before heating

$$V = \frac{P_0V_0}{P}(1 + \alpha t_1),$$

after heating

$$V + v = \frac{P_0V_0}{P}(1 + \alpha t_2). \qquad (278)$$

The increase in volume v, is therefore

$$v = \frac{P_0V_0}{P}\alpha(t_2 - t_1) = \frac{P_0V_0}{P}\alpha t \qquad (279)$$

and by (277) and (256)

$$c_p - c_v = \frac{P_0V_0}{M}\alpha = \frac{R}{M}\,\frac{ergs}{gram\text{-}degree} \qquad (280)$$

You have ordinarily not used a

PV and R are usually given in mechanical units, hence, if we wish to express $c_p - c_v$ in thermal units, the right hand member of the equation must be divided by J,

or
$$c_p - c_v = \frac{P_0 V_0}{JM} \alpha = \frac{P\alpha}{Jd_0} = \frac{R}{JM} \frac{\textit{calories}}{\textit{gram-degree}} \tag{281}$$

where d_0 is the density at 0° C.

Since the specific heats s_p and s_v are numerically equal to c_p and c_v,

$$s_p - s_v = \frac{P_0 V_0}{JM} \alpha \text{ numerically} \tag{282}$$

184. Coefficients of Volume Elasticity. From Fig. 95 it appears that during an ***adiabatic compression*** the change in ***pressure*** must be ***larger*** to produce a given change of volume than in the case of an ***isothermal compression.*** From the definition of the coefficient of volume elasticity (Art. 58)

$$e = \frac{dp}{dV} \cdot V$$

it is seen that a distinction must be made between the adiabatic and isothermal coefficients, e_a and e_i. Let the changes in pressure corresponding to a small change of volume

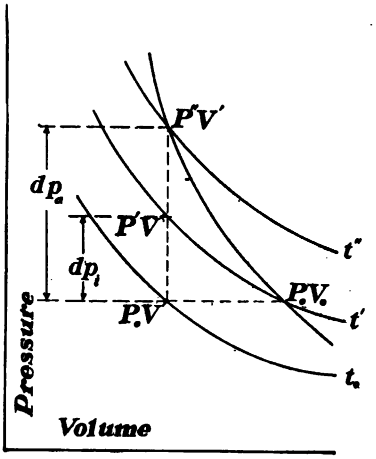

FIG. 96.

be $\quad V_0 - V' = dV$

$\quad P'' - P_0 = dp_a$

and $\quad P' - P_0 = dp_i$

respectively (Fig. 96), then, as e_a and e_i vary as dp_a and dp_i respectively, dV being constant,

$$\frac{dp_a}{dp_i} = \frac{e_a}{e_i} \tag{283}$$

Let the corresponding changes of temperature $t'' - t_0$ and $t' - t_0$ be dt_a and dt_i. But dp_a and dp_i may also be considered as the increase in pressure of the gas when heated under constant volume, V', through dt_a and dt_i degrees. tr ?

Since by Art. 167 (eq. 254),

$$P_0 + dp = \frac{P_0 V_0}{V} \cdot (1 + \alpha dt)$$

the increases in pressure, dp_a and dp_i, are proportional to the increases in temperature, dt_a and dt_i, or by (283)

$$\frac{e_a}{e_i} = \frac{dt_a}{dt_i} \tag{284}$$

Now consider the gas to be brought from $P_0 V'$ to $P'' V'$ by two different methods: (1) by heating under constant volume, for which the heat needed is $Mc_v dt_a$; (2) by heating under constant pressure, until its volume has become V_0, in which case the heat needed is $Mc_p dt_i$. Then compress the gas adiabatically until it reaches the state $V'P''$. No heat from the outside is needed for this. During the two parts of the last process, work is done by the gas during the expansion, and done *upon it* during compression; but if the increment of pressure be *very small* in comparison with the total pressure, the total external work may be considered as practically zero, in comparison with the heat involved.

The total change in energy of the gas along the two paths s may, therefore, be set equal to each other and equal to the heat absorbed, or

$$Mc_v dt_a = Mc_p dt_i \tag{285}$$

If we call the ratio of the two specific heats γ,

$$\frac{e_a}{e_i} = \frac{dt_a}{dt_i} = \frac{c_p}{c_v} = \frac{s_p}{s_v} = \gamma \tag{286}$$

185. Velocity of Sound in a Gas. As has been shown (Art. 111), the velocity V, of sound in a gas is given by

$$V = \sqrt{\frac{e}{d}}$$

But, since the compressions and rarefactions in the air occur under adiabatic conditions, the corresponding coefficient of volume elasticity must be used, or

$$e = e_a = \gamma e_i \tag{287}$$

and the expression for the velocity of sound becomes, since e_i is equal to the pressure P (Art. 113),

$$V = \sqrt{\gamma \frac{P}{d}}$$

The values for γ for several different gases under a pressure of one atmosphere are given in Table IX, on page 214.

186. The Joule-Thomson Effect. In the experiments described (Art. 181), Joule found no change of energy in a gas due to change of volume alone. If this were true, no forces would exist between the separate molecules of the gas, and the energy of the gas would simply be the sum of the kinetic energies of its molecules.

Joule and Thomson (Lord Kelvin) found, however, by their famous "porous plug" experiment[1] that there is a slight change in energy when a gas expands. They passed gas slowly from a vessel at high pressure into the open air through a plug made of cotton wool. The temperature of the gas was measured just before entering and after leaving the plug, and it was found that in most gases the temperature after expansion was lower than before. In the case of hydrogen a heating was observed. It has since been shown, however, that even hydrogen gas will cool if the original temperature of the gas under high pressure be below – 80° C. The compressor, producing the high pressure, does work upon the gas, equal to the product of the pressure of its piston into the change of volume. On the other hand, the gas at the lower pressure side does work, equal to the product of its volume into this lower pressure. For a perfect gas these two amounts of work should be equal.

[1] *Phil. Mag.*, 1852.

Of course we must consider the fact that Boyle's law does not hold strictly for ordinary gases, and that consequently the work Pv, done upon the gas at the high pressure does not quite equal the work done by the gas on the low pressure side. However, after taking this into account, the porous plug effect, which in all cases is quite small, indicates that there is some intermolecular action in all ordinary gases, and that, in general, energy is required to produce a mere expansion.

CHAPTER XXIII

TRANSFORMATION OF HEAT INTO MECHANICAL ENERGY

187. Modes of Transformation. The transformation of mechanical energy or of any other form of energy into heat is of common occurrence and no difficulty is encountered in making such transformation complete. On the other hand, no method has ever been devised for reversing completely any process which has produced heat, although — according to the first law of thermodynamics — this would seem to be theoretically possible.

In fact, it has been found that a *continuous* transformation of heat into mechanical energy is possible, only under the condition that at the same time heat shall be transferred by the working substance passing through the engine which performs the work, from a higher to a lower temperature, or, in other words, that the engine shall take in a certain amount of heat at a high temperature and give out a smaller amount of heat at a lower temperature.

188. Carnot's Cycle. A process, first described by Carnot, will illustrate the statement of the last article. It is a cycle, consisting of four parts.

1. A gas, kept in constant contact with a reservoir at temperature t_1°, is expanded isothermally from P_1V_1 to P_2V_2, as represented (Fig. 97) by the curve AB. It absorbs an amount of heat H_1. External work, equal to the area $ABba$ is done by the gas.

2. The gas expands adiabatically, until its temperature has fallen to t_2. No heat is added during this step. The external work done by the gas is represented by area $BCcb$.

3. The gas is compressed isothermally, while in constant contact with a cool reservoir at temperature t_2°. An amount

of heat H_2 is given out, and the work done upon the gas is equal to the area *CDdc*.

4. The gas is compressed adiabatically until it again reaches its original condition, being heated to t_1. No heat is given out and the external work done upon the gas is represented by the area *DAad*.

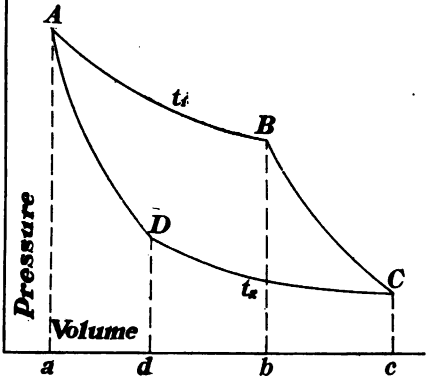

FIG. 97.

During the whole cycle external work, equal to the area $ABCD = ABba + BCcb - CDdc - DAad$, is done by the gas. This work has been obtained by a transformation of a part of the heat H_1 entering the engine. Therefore

$$H_1 - H_2 = W \qquad (288)$$

The *efficiency* of the cycle is the ratio of the useful work to the total energy H_1 put into the machine. For this "theoretical" engine the efficiency may be shown to be

$$\frac{W}{H_1} = \frac{H_1 - H_2}{H_1} = \frac{T_1 - T_2}{T_1} \qquad (289)$$

where T_1 is the absolute temperature of the hotter reservoir.

189. Irreversible Processes. While the work done by the engine might afterwards be used to restore some heat to the hotter reservoir, as, for example, by working the above cycle backwards, still there are some common processes in nature in which heat passes from higher to lower temperature without doing work. Such a process cannot be reversed. The most important example of this kind is the conduction of heat, as through the walls of the cylinder of a steam engine. After the heat has once reached the lowest temperature of all the surrounding bodies, it is impossible to obtain any further useful work from it.

Since conduction of heat cannot be avoided in steam engines, and since the heat thus transferred is lost, so far as useful work is concerned, it is evident that the efficiency of our actual steam engines must be smaller than that of the Carnot cycle, where no such loss was assumed to occur.

190. The Reciprocating Steam Engine. The first successful attempt to utilize the energy of steam was due to James Watt (1736–1819), whose apparatus took the form of the reciprocating engine. Figure 98 illustrates the action of such an engine. Its essential parts are (*a*) the cylinder C_1C_2, in which a piston P moves back and forth; (*b*) a steam chest S, from which the steam, at high temperature, enters the cylinder, through openings A and B, called the ports; (*c*) the exhaust e, through which the steam, at a lower temperature, leaves, escaping either into the air or into a vacuum chamber, called the condenser.

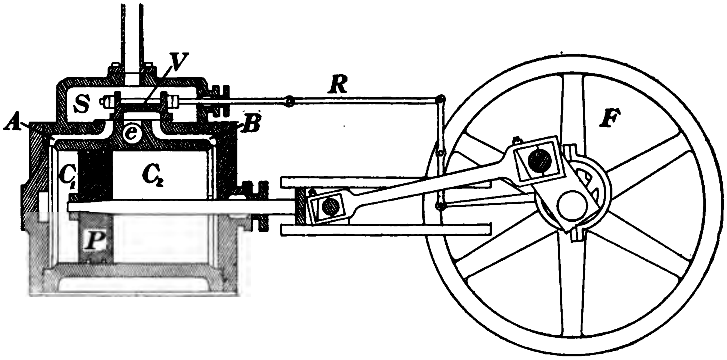

FIG. 98.

A slide valve V is moved back and forth by the eccentric rod R, connected with the flywheel F, or some other rotating part of the engine. The motion of this valve is such that it connects the port on one side of the cylinder with the steam chest, and a short time afterwards the other port with the exhaust. With the position of the parts of the engine as shown in the figure, the pressure of the steam, entering from the boiler, moves the piston to the right and sets the flywheel in motion. Shortly before the piston reaches its extreme position, the slide valve has moved far enough to the left to close both ports; the steam at the right-hand side is now compressed and its temperature rises. But in the meanwhile the slide valve has moved so far to the left that the steam will now enter through port *B* while the left cylinder is connected with the exhaust. Now the whole process is reversed, the piston returns to its original position and the cycle is repeated. The reciprocating action of the piston keeps up the motion of the rotating part of the engine.

In some engines the process of expansion from the high pressure of the boiler to that of the exhaust is distributed over a number of steps, each successive cylinder being larger in diameter than the preceding one. Such engines are called compound, triple-expansion or quadruple-expansion engines according as the number of steps is two, three or four.

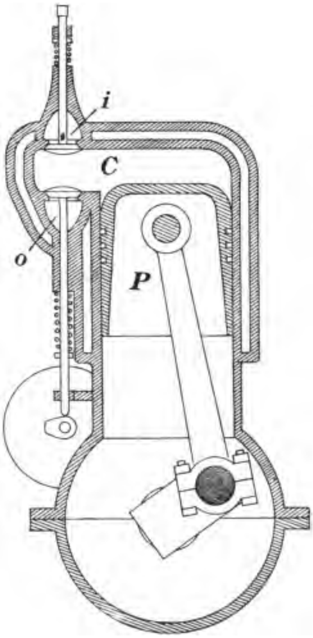

Fig. 99.

191. The Internal Combustion Engine. The best known engines of this type are the gas and gasoline engines. Instead of leading steam from a separate boiler into the cylinder, a mixture of air and gas, or of air and gasoline which evaporates in the cylinder, is passed into the cylinder and there exploded. This produces the high temperature and the pressure necessary to push the piston forward. The cylinder is supplied with two valves, the inlet and the outlet valves, *i* and *o* (Fig. 99).

Both valves are closed when the explosion takes place producing the expansion stroke (1), which drives the piston forward. On its return (2), the outlet valve opens and the burnt gases are ejected, after which this valve closes. During the suction stroke (3), the inlet valve opens and the fuel enters the cylinder. With both valves closed the piston returns, making a compression stroke (4), and compresses the gas mixture. Then the whole cycle repeats itself.

It is seen that four strokes or two complete to-and-fro motions of the piston follow each explosion. In its simplest form such an engine requires but a single cylinder. However, in order to increase the available power as well as to minimize the jar, due to the explosions, such engines are now furnished with two, three, four, and even six cylinders, all geared to the same shaft and delivering the thrusts, due to their individual explosions, at symmetrically periodic intervals. The action of such engines is characterized by remarkable speed and smoothness.

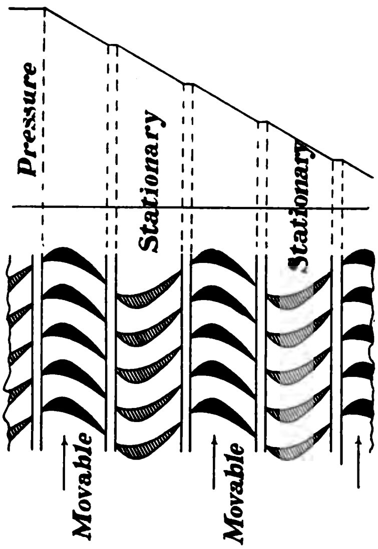

Fig. 100.

192. The Steam Turbine. The steam turbine consists of revolving drum on whose periphery a large number of vanes are mounted (Fig. 100). Jets of steam are directed agains these blades and by their impulses produce the rotation of the wheel. In the simplest types the transformation of heat into kinetic energy of the steam takes place at the nozzles through which the steam enters the turbine and the kinetic energy of the steam produces the useful work.

In the "multistage" turbine the expansion is divided into steps, the blades being further apart in successive stages. In these engines the steam, which is still under considerable pressure when striking the first set of vanes, also does work by its expansion as it passes through the turbine, and by this method produces work inside the engine at the expense of the heat which it contains.

The angular velocity of steam turbines is very high, although it is lower in the multistage type than in the simple turbine. These machines occupy less space and run more quietly than the reciprocating engines and are especially adapted for driving alternating current dynamos and centrifugal pumps. On account of the absence of jarring they are coming more and more into use on board steamships.

Problems

1. What is the thermal capacity of 1 cu ft of air, expressed in British thermal units, the specific heat of air being 0.24 and the density 0.08 lb per cubic foot?

2. A piece of copper, weighing 300 g and heated to 99°.4 C, is plunged into 400 g of water contained in a copper calorimeter whose mass is 90 g. The temperature of the calorimeter and its contents is raised from 20° to 25°.1 C. Find the specific heat of copper and the thermal capacity of the piece of copper introduced into the calorimeter.

3. To find the temperature of a certain furnace, a piece of platinum of mass 10 g is placed in it. After taking the temperature of the furnace it is suddenly plunged into 40 g of water at 10° C. The temperature of the water rises to 24° C. What is the temperature of the furnace, assuming the specific heat of platinum to be 0.032?

4. Calculate the thermal capacity of unit volume of mercury and of glass, the density of the latter being 2.5 g/cm^3. Show that the thermal capacity of the immersed part of the thermometer may be taken without an appreciable error as numerically equal to 0.47 of its volume.

5. One gram of anthracite coal if burned produces 7800 calories. How much heat, expressed in British thermal units, is produced by the burning of 1 lb of coal?

6. How much heat is necessary to heat the air in a certain room, 6 × 5 × 3 m from 0° C to 25° C? How much water, cooling from 100 to 25° C, would furnish the heat required?

7. If it require half a horse power for 2 min to drill through a block of iron of 800 g mass, how much heat is produced? Supposing nine tenths of the heat to appear in the iron, how much does the temperature of the block rise?

8. The falls of Niagara are 160 ft high. How much warmer should the water be at the bottom than at the top?

9. What must be the speed of a lead bullet if, upon striking a target, its temperature be raised from 27° C to its melting point, 327° C? Assume that all the heat produced serves to heat the bullet, and that the specific heat of lead is 0.031.

10. When a street car weighing 4000 kg and having a speed of 20 km per hour is stopped by the brakes, how much heat is produced?

11. Suppose the earth's rotation around its axis to be suddenly stopped. What change in temperature would be produced? Consider the earth a sphere of uniform density, and of specific heat 0.2; radius of earth, 6360 km. The moment of inertia of a sphere around any diameter is $2/5 MR^2$, where M is its mass and R the radius.

12. If the average pressure in the cylinder of a steam engine be 10 kilograms-weight per square centimeter, and the area of the piston be 300 cm², how many calories does the steam lose when it pushes the piston 50 cm forward?

13. How much heat is needed to produce the work done by a 100 H. P. engine running for one hour?

14. The efficiency of a condensing engine is about 16 per cent. How much coal is consumed by a 20,000 H. P. condensing engine in one hour, assuming 30 per cent of the heat of combustion to be lost in passing from the coal to the engine? (See problem 5.)

15. The average locomotive has an efficiency of about 6 per cent. What horse power does it develop when consuming 1 ton of coal per hour? (See problem 5.)

16. A perfect engine takes steam from a boiler at 150° C, and exhausts into a condenser at 30° C. Compute its efficiency.

17. The mixture of air and gas in a gas engine reaches a temperature of about 1100° C when it is ignited, and the temperature of this mixture is reduced to 600° C by expansion. What would be the efficiency of a perfect engine working between these temperatures?

18. If a compound marine engine consume 2 lb of coal per horse power every hour, what per cent of the energy of the coal is being transformed into work in the cylinder?

19. Calculate from the specific heats of air, and, using equation (281), the value of the mechanical equivalent of heat. Density of air at 0° C = 0.001293 g per cm³. This calculation was first made by Mayer in 1842.

CHANGE OF STATE

CHAPTER XXIV

FUSION

193. The Melting Point. It has already been shown that when the temperature of a solid reaches a certain point the body begins to melt, and that any further addition of heat, if not too rapidly applied, simply serves to hasten the melting process, without changing the temperature. If the process be interrupted by preventing heat from reaching or leaving the mixture formed, the temperature will remain constant and no further change in the relative amounts of solid and liquid takes place, thus showing that in this condition stable equilibrium exists.

The *melting point* or the *fusing point* of a substance is therefore *that temperature at which the solid and liquid states are in equilibrium under the existing pressure.* The fusing point is usually referred to atmospheric pressure. Above this temperature the substance exists as a liquid, while below this temperature it is usually a solid. When the change of state occurs at a relatively low temperature, the substance in question is generally known in the liquid state and we consequently speak of the temperature of transition as the freezing point. From the definition it is clear that *the freezing point and the melting point are one and the same temperature.*

Only crystalline bodies have definite melting points. Amorphous substances, such as glass, paraffine or wax, grow more and more plastic as the temperature is raised, and finally become liquid, hence no definite temperature can be found at which a transition from the distinctly solid state to the distinctly liquid state occurs. On account of this gradual change in plasticity such substances may be heated to softness and may then be molded into any desired shape, or two pieces may even be welded together. By means of polarized light it may be shown that glasses soften sufficiently to permit of some molecular motion, and the removal of internal strains, at temperatures from 250° to 300° below the point at which the same glasses become fluid.

TABLE X

MELTING POINTS UNDER ATMOSPHERIC PRESSURE

SUBSTANCE	MELTING POINT	SUBSTANCE	MELTING POINT
Mercury	−38°.8	Aluminium	657°
Phosphorus	44°.3	Silver	961°
Sulphur	115°	Gold	1063°
Tin	232°	Copper	1084°
Bismuth	260°	Iron	1100°
Cadmium	320°	Steel	1350°
Lead	327°	Platinum	1778°
Zinc	419°	Iridium	2200°

194. Heat of Fusion. The quantity of heat necessary to melt a body of mass M is proportional to its mass, consequently we have

$$H = LM \tag{290}$$

The proportionality factor

$$L = H/M \tag{291}$$

is called *the heat of fusion of the substance* and may be defined as the *calories per gram needed to change the substance from the solid to the liquid state, without change of temperature.* The heat of fusion is therefore a measure of the energy needed to produce this change of state. It is numerically equal to the heat absorbed in the fusion of one gram of the substance. It may easily be found by the method of mixtures. For water the value of L is nearly 80 calories per gram.

The same amount of heat, LM calories, is liberated when a mass M *of the same liquid freezes.* Pails filled with water are often placed in cellars in order that the water in freezing may liberate enough heat to prevent the freezing of the fruit or vegetables in the cellar. Ice keeps a refrigerator cool because it absorbs from the refrigerator and its contents the heat needed to melt the ice.

This absorption of heat during melting is employed in *freezing mixtures*. If salt be mixed with ice, a salt solution is produced. The heat of fusion of the ice must be supplied by the mixture and the vessels standing in it. Melting is therefore a cooling process, and freezing a warming process.

195. Supercooling. While a solid when heated always begins to melt as soon as the melting point is reached, a liquid, if carefully protected from mechanical disturbances, may frequently be cooled below this point without freezing. Thus water may easily be cooled to − 10°, or more, without the formation of ice. This phenomenon is called supercooling. The liquid is then in a state of equilibrium less stable than that of the mixture of solid and liquid which will ensue as soon as a crytal is brought into the liquid. S

This is easily shown with hyposulphite of soda. The salt, after having been melted at a temperature somewhat above 50°, will remain liquid in a stoppered flask for an indefinite time at ordinary temperatures. But the moment a crystal is dropped in, fine needles will be seen to shoot from it in all directions and soon the whole volume is filled with crystals. At the same time the temperature rises to the normal melting point, 49°.5. This evolution of heat is due to the heat of fusion liberated during the crystalization.

With some substances supercooling may be continued to such relatively low temperatures that the liquid becomes more and more viscous and finally solid, so far as its mechanical properties are concerned. Ordinary glass is an example of such a substance. Quartz crystals melt at about 1500°. If the molten mass be cooled, it becomes an amorphous, transparent substance, of valuable physical properties (Art. 155). Quartz glass, or fused quartz, is therefore nothing but supercooled liquid quartz.

196. Change of Volume during Fusion. In general, substances contract when they freeze and expand when they melt. Notable exceptions, however, are cast iron and type metal. While other bodies would, upon solidification, draw away from the mold, these metals expand, press into the mold, and reproduce the finest and most minute details.

The most common exception is water. One gram of water expands upon freezing from 1.00012 cm^3 to 1.092 cm^3. This expansion plays an important rôle in the disintegration of rocks during the winter season. The pressure exerted by freezing water is very great. A cast iron bomb if filled with water and securely sealed by a screw cap will, when placed in a freezing mixture, explode with a loud report.

197. Influence of Pressure upon the Freezing Point. All substances which expand upon melting have their melting points raised by an increase of pressure; all substances which contract upon melting have their melting points lowered. Or, in general, *pressure favors that state in which the volume is least.* This influence of pressure is very small. Thus an increase of pressure of one atmosphere lowers the melting point of ice only 0°.0075 C.

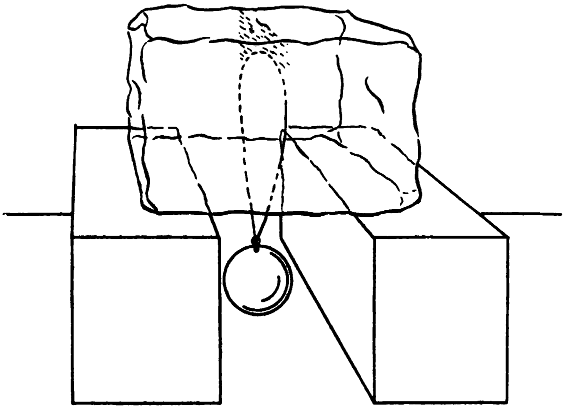

FIG. 101.

Snow easily packs under pressure if the temperature be near the freezing point, but will not do so if the weather be too cold. Two pieces of ice may be frozen together under warm water by applying considerable pressure and then releasing them suddenly.

Let a mass of several kilos be hung by a thin wire over a block of ice (Fig. 101). In a short time the wire will be found to have cut completely through the block, leaving the ice as solid as it was at the beginning. The ice just below the wire melts on account of the increased pressure and it absorbs heat of fusion by which a temperature a little below 0° is produced at the place where the wire touches the ice. The water passes the wire and freezes again above it, on being released from the pressure. If the mass be of metal, it will be better to insert a link of string between the mass and the fine wire.

198. Freezing Point of Solutions. A small quantity of salt dissolved in water lowers the freezing point nearly in proportion to the quantity dissolved. When such a solution begins to freeze, however, we find that it is only the *water* which freezes out, while the salt, remaining in solution, makes it more concentrated and consequently lowers the freezing point still further.

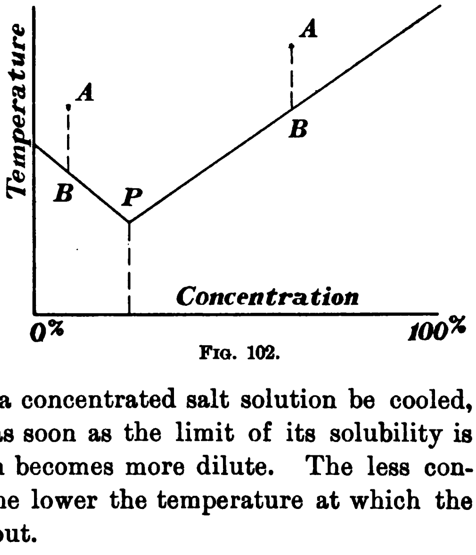

FIG. 102.

If, on the other hand, a concentrated salt solution be cooled, the *salt* crystallizes out as soon as the limit of its solubility is reached, and the solution becomes more dilute. The less concentrated the solution, the lower the temperature at which the salt begins to " freeze " out.

In the first case the solution becomes saturated with respect to the solvent, water; in the second, with respect to the dissolved substance, salt. If the temperature at which saturation occurs be plotted as a function of the concentration of the solution, two curves (Fig. 102) are obtained which meet at a point, P, of minimum temperature. Cool a solution of any concentration, for example one corresponding to a point A. Crystallization, either of the salt or of the solvent, will begin as soon as the temperature has fallen to that of some point B on the saturation curve. Upon further cooling, the solution moves along the saturation curve to the point *P*, where the whole mass solidifies as a *mechanical* mixture of ice and salt. Such a mixture is called a *cryohydrate*. The cryohydrate of common salt and water contains 26 per cent of salt and solidifies at – 23° C.

Alloys of metals behave in a manner similar to solutions. In general, the melting point of the solvent is lowered by the addition of a small quantity of some other metal. Solders are well known examples of this fact.

A notable example of this is found in Wood's metal, composed of 52 per cent of bismuth, 26 per cent of lead, 15 per cent of tin, and 7 per cent of cadmium. This alloy melts at 75°.5 C, so that a spoon made of this metal melts when placed in hot tea.

CHAPTER XXV

VAPORIZATION

199. Vaporization. Vaporization is the process of transforming a substance from the solid or the liquid state into the gaseous state. According to the molecular theory some of the molecules of a solid or of a liquid possess sufficient kinetic energy to carry them through and beyond the surface of the mass. The molecules thus "freed" form the vapor filling the surrounding space. By virtue of their impact upon the restraining walls they exert a definite pressure. This pressure is termed the vapor pressure of the substance for that temperature.

Vaporization from a liquid is called *evaporation.* Vaporization from a solid is termed *sublimation.*

200. Evaporation. If a small amount of liquid be introduced into a barometric tube above the mercury, it begins to evaporate and, on account of the pressure which the vapor exerts, the mercury falls to a certain height which is independent of the width of the tube and of the space filled with vapor, so long as there is any liquid left in the tube (Fig. 103). Vapor which is in equilibrium with its liquid is called *saturated vapor.* The molecular theory teaches that after equilibrium is established, as many molecules leave the liquid during a given time as reënter it from the vapor. If the space above the liquid be increased, more vapor is formed, but so long as any liquid is present, the meniscus of the mercury in the tube remains at the same height above the level of the mercury in the cup. This shows that *saturated vapor at a given temperature always exerts the same pressure.*

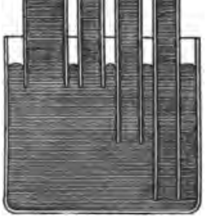

FIG. 103.

If the volume be increased until all liquid is evaporated, the vapor becomes isolated and the pressure at a given temperature depends only upon the volume occupied by the vapor. It is then called an unsaturated or superheated vapor and follows very nearly the gas law.

In order to distinguish the pressure exerted by an unsaturated vapor from that of a saturated vapor we shall call the pressure due to the latter *vapor tension.*

201. Evaporation and Dalton's Law. When water is introduced into a barometric tube at a temperature of 20° C, the mercury meniscus falls 1.74 cm. The vapor tension of water at 20° corresponds, therefore, to 1.74 cm of mercury, or it is equal to 23,170 dynes per square centimeter. If the tube had been partially filled with some gas which does not act chemically upon water, the lowering of the mercury upon the introduction of water would have been very nearly the same as before. This shows that the vapor tension of the water has simply been added to the pressure of the gas previously in the tube. This illustrates Dalton's law (Art. 95).

A liquid evaporates into a vacuum or into any mixture of gases until the individual pressure produced by its vapor equals the vapor tension of the liquid at the existing temperature. The only influence of the other gases will be a decrease in the rate of evaporation. Thus water at 20° C evaporates so long as the partial pressure of water vapor in the surrounding atmosphere is below 1.74 cm of mercury, or until the atmosphere becomes saturated with vapor. Since water vapor has, under the same conditions, a smaller density than dry air, it follows that at the same temperature and under the same pressure a given volume of dry air weighs more than the same volume occupied by air so "laden with moisture." For this reason the barometer reading is low when the air contains much water vapor.

202. The Vapor Tension Curve. Change in temperature has a great influence upon the pressure exerted by a saturated vapor. The relation between the two may be plotted as a curve on a pressure-temperature diagram (Fig. 104). The points on this curve will then represent the condition of equilibrium between the liquid and its saturated vapor. Such a curve is called the *vapor tension* or *evaporation curve*. If the pressure of the vapor in contact with its liquid be greater than the vapor tension of the liquid at the same temperature, the vapor will condense. If the pressure be smaller, the liquid will evaporate. For conditions of equilibrium, therefore, the space on the left-hand side of the curve represents the liquid state alone, while that on the right represents the gaseous state. Figure 104 represents these relations for water.

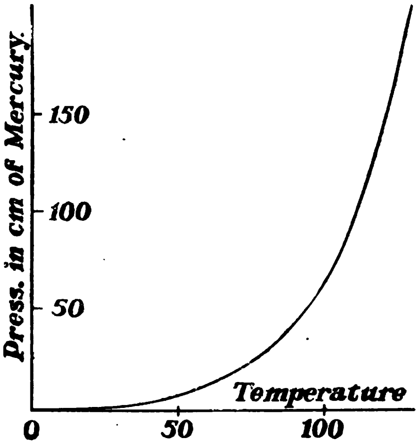

FIG. 104.

203. The Boiling Point. When a liquid is heated, its temperature rises and its vapor tension accordingly increases, until finally bubbles are formed in the liquid itself, especially at the place where the heat is applied. The liquid "boils" when evaporation no longer takes place quietly on the surface. The bubbles of saturated vapor expand against the pressure of the surrounding atmosphere and that of the small layer of liquid above them as they rise to the surface, while at the same time the liquid rapidly evaporates *into* the bubble from all sides.

The boiling point of a substance is the temperature at which the vapor tension of the substance equals the gas pressure upon the liquid, no matter to what this pressure may be due. The "normal" boiling point is always referred to a pressure corresponding to 760 mm of mercury.

The boiling point is evidently the temperature on the vapor tension curve corresponding to the gaseous pressure on the liquid. It is given accurately by the reading of a thermometer hung in the vapor a short distance above the boiling liquid.[1]

TABLE XI

BOILING POINTS OF SOME LIQUIDS UNDER ATMOSPHERIC PRESSURE

SUBSTANCE	BOILING POINT	SUBSTANCE	BOILING POINT
Ethylene	– 103°	Chloroform . . .	+ 61°
Ammonia	– 38°.5	Alcohol	+ 78°
Chlorine	– 33°.6	Toluene	+ 110°
Ether	+ 35°	Glycerin	+ 290°
Carbon bisulphide . .	+ 46°	Mercury	+ 357°

204. Superheating. The vapor bubbles form in a boiling liquid usually at places where minute air bubbles adhere to the walls of the vessel or to some foreign substances present in the liquid. After the air has been removed by previous boiling, the liquid may often be heated considerably above the boiling point, since no opportunity is given for the formation of vapor inside the liquid. The liquid is then said to be superheated. In such cases sudden boiling will finally set in with almost explosive violence.

In order to show the superheating of a liquid, heat a beaker full of water that has been boiled for a few minutes and allowed to cool. The temperature may be carried several degrees above the boiling point as shown by the reading of the barometer. If now there be added to the water a small quantity of white sand or finely powdered glass, violent boiling will ensue.

As another illustration of superheating, fill a tube closed at one end and about 80 cm long, with mercury and a few cubic centimeters of ether. Invert it in a deep dish filled with mercury. If care be taken to remove all air, the ether will remain liquid on the top of the mercury. The experiment succeeds often with a mercury column more than 76 cm long, the liquid being actually under a pull and yet not evaporating. A slight jar, however, will start evaporation and the mercury will rapidly fall to a position, corresponding to the normal pressure given by the vapor tension curve ; that is, 44 cm at 20° C.

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[1] For method of determination of the boiling point see Manual, Exercise 32.

A superheated liquid is therefore in a less stable state than the mixture of the liquid and its vapor at the same temperature.

205. Influence of Pressure upon the Boiling Point. If a vessel with water at ordinary temperature be placed under the receiver of an air pump, a few strokes of the pump will cause the water to boil. The experimental result that the boiling point is lowered by a decrease of pressure could have been predicted from a study of the general shape of the vapor tension curve and from the definition of the boiling point as given (Art. 203).

The influence of pressure upon the boiling point is much more marked than its influence upon the melting point. The boiling point of water, for example, changes 0°.37 for a change of 1 cm of mercury in barometric pressure.

Since the change of barometric pressure amounts to about 1 mm of mercury for a vertical rise of 11 m, water boils on Pike's Peak, 4310 m above sea level, at 85°, a temperature at which many ordinary cooking operations are impossible. On the other hand, the boiling point of water in a steam boiler under a pressure of 100 lb per square inch is 155° C.

TABLE XII

BOILING POINT OF WATER UNDER DIFFERENT BAROMETRIC PRESSURES

PRESSURE	BOILING POINT	PRESSURE	BOILING POINT	PRESSURE	BOILING POINT
73.0	98°.88	75.0	99°.63	77.0	100°.37
73.5	99°.07	75.5	99°.82	77.5	100°.55
74.0	99°.26	76.0	100°.00	78.0	100°.73
74.5	99°.44	76.5	100°.18	78.5	100°.91

206. Vapor Tension and Boiling Point of Solutions. The vapor tension of a solution containing a non-volatile salt is always lower than that of the solvent at the same temperature. For dilute solutions the decrease is proportional to the concentration. The vapor tension curve of a solution, therefore, always lies to the right of that of the solvent. In Fig. 105 the vapor tension of a solution is indicated by the dotted lines, while the full line gives the vapor tension of the solvent. Consequently for a given pressure, the boiling point *M* of the solution is higher than *N*, that of the solvent. For example, 35.5 parts of sodium chloride (common salt) dissolved in 65.5 parts of water lowers the vapor tension at 100° nearly 18 cm of mercury and raises the boiling point to 107°.5.

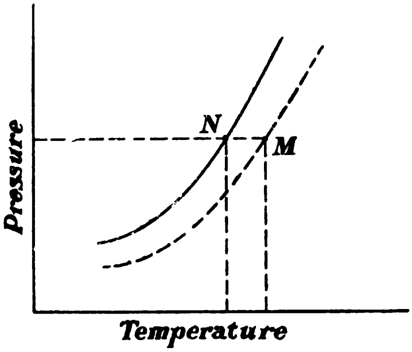

FIG. 105

It is frequently stated that the temperature of the vapor above the boiling solution is the boiling point of the solvent. A thermometer hung in the vapor condenses some of the vapor, and then indicates the boiling point of the solvent. But if such condensation be prevented, it may be shown that the vapor of a boiling solution is at a temperature equal to the boiling point of the solution and not to that of the solvent.

When a solution, containing non-volatile substances is boiled, only the solvent evaporates, while the dissolved substance crystallizes out, when the solution becomes sufficiently saturated.

207. Distillation. The vapor rising from a boiling liquid may be condensed into a liquid by being passed into a cold vessel. This combination of boiling and condensation is called distillation. It affords a convenient method for freeing a liquid from impurities, as water from salts, or mercury from other metals. Evaporation in a vacuum is employed either when the normal boiling point is very high as in the case of mercury, or when the substance crystallizing out is chemically changed at the normal boiling point. Sugar solutions are evaporated in a vacuum to prevent scorching the sugar at the boiling point of the syrup.

The saturated vapor above a mixture of two volatile liquids, such as alcohol and water, contains in general the two constituents in a different proportion from that in the solution. If such a solution be boiled, the constituent whose percentage is higher in the vapor than in the liquid, distills over more rapidly and consequently is found in much larger concentration in the distillate than in the original solution.[1]

In some mixtures there exists a definite concentration at which the proportion of the constituents is the same in both, liquid and vapor. Solutions of this concentration distil over unchanged. An example of this is common alcohol of 96 per cent concentration, which boils at 78°.17 C, a temperature 0°.13 C below the boiling point of pure alcohol. At higher concentration than 96 per cent, there is relatively more water in the vapor than in the liquid, and, consequently, more water distills over than alcohol.

It is evident, that alcohol containing much water may be concentrated by repeated distillation, up to a strength of 96 per cent, but that it is impossible to obtain by this method an alcohol of greater concentration. Absolute alcohol is obtained by allowing 96 per cent alcohol to stand for some time over quicklime.

208. Heat of Vaporization. From the foregoing considerations it is clear that when a liquid is changed to a vapor, a certain quantity of heat is needed to effect this transformation. *The heat of vaporization of a substance denotes the calories per gram needed to vaporize that substance without change of temperature.* This heat of vaporization is constant for a given substance for a given temperature, but *decreases* as the temperature *increases.* For water this relation is given by the equation, proposed by Griffiths,

$$L = 596.6 - 0.601\,t \qquad (292)$$

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[1] For a discussion of the behavior of different mixtures of two volatile liquids, see Chwolson, *Lehrbuch der Physik*, vol. III, p. 934.

More recent determinations are found in the following table:[1]

TABLE XIII

HEAT OF VAPORIZATION OF WATER

TEMPERATURE	L IN CAL. PER GRAM
0° C	596.3
25° C	582.5
50° C	568.2
75° C	553.3
100° C	538.0

Since water, on vaporization at 100°, expands to about 1650 times its liquid volume, it follows that by this expansion it has done work and has absorbed energy. Of the total heat energy absorbed in the vaporization of water, about $\frac{1}{13}$ is needed to effect this expansion, while the remaining $\frac{12}{13}$ is to be regarded as an increase in the potential energy of the water molecules, so long as they exist in the form of steam. When the steam condenses again to water, all the heat absorbed during vaporization is given out. Upon this principle depend many important industrial applications, such as steam heating, steam cooking, etc.

209. Cooling by Evaporation. Owing to the large amount of heat absorbed, rapid evaporation is a very efficient means of cooling. To that end we sprinkle the floors or sidewalks on a hot day, or bathe with water or alcohol, patients suffering from fevers. Water kept in a porous jar is always at a lower temperature than that of the surrounding air, owing to the rapid evaporation of part of the water on the outer surface of the porous vessel.

Water may even be frozen by its own evaporation provided this be made sufficiently rapid. To this end a small flat cup of water, thermally insulated, is placed over a shallow dish containing concentrated sulphuric acid (Fig. 106), and the whole covered by a flat receiver upon the plate of an air pump. At the first stroke of the pump a cloud of mist is seen which is largely absorbed by the acid. After the air has been pumped out of the water, the pressure in the receiver is soon reduced to a value below the vapor tension of water at room temperature, and rapid boiling begins. Now, since all heat needed for the vaporization must be supplied from the cup and its contents, it is evident that the temperature of the water must fall quickly, and if the pump be worked rapidly, the water boils and freezes at the same time. For success in this experiment the pump must not only exhaust rapidly, but must also reduce the pressure in the receiver to something less than 4.6 mm of mercury (Art. 211).

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[1] A. W. Smith, *Monthly Weather Review*, October, 1907.

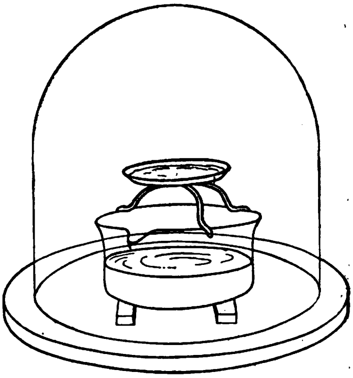

FIG. 106.

210. Cooling by Expansion of Gases. We have already seen (Art. 175) that if a gas be heated under constant pressure, it expands, and absorbs heat. The converse of this truth is seen in the fact that if a gas under pressure be allowed to expand it tends to absorb heat, and its temperature rapidly falls. If the gas be liquified and held under great pressure, then on its release we have the combined cooling effects due to vaporization of the liquid and the expansion of the resultant vapor. In this way a gas may be cooled so suddenly as to be frozen solid.

If a cylinder containing liquid carbon dioxide be placed in a vertical position with the valve down, then on opening the valve, the liquid will be driven out by the pressure of the confined gas. Owing to the high vapor tension of the liquid, a very rapid evaporation and expansion occurs, and the gas is quickly chilled to the freezing point. If a bag made of flannel or chamois skin be held over the opened valve, it will soon be filled with a snowy substance, the solid carbon dioxide, which under atmospheric pressure has a temperature of −78°. If this snow be gathered up and compressed into a brick, it may be kept for hours in the open air. It slowly sublimes without passing through the liquid state (Art. 211).

A mixture of solid carbon dioxide and ether or alcohol is more convenient for experimentation than the solid dioxide alone, because the liquid at −78° insures better thermal contact with bodies immersed in it. With this mixture mercury may readily be frozen.

In commercial refrigerating plants liquid ammonia is rapidly evaporated in a system of coils by the action of a pump which constantly draws off the ammonia vapor, and at the same time compresses it at another part of the apparatus, the condenser, which is cooled by running water. This takes up the heat of condensation. Under the high pressure in the condenser the ammonia liquefies and then by a regulating valve is slowly readmitted to the coils in which the evaporation takes place. This part is called the evaporator and is usually immersed in a tank filled with brine. The brine is kept in circulation by separate machinery. In artificial ice plants it flows around the vessels containing the water to be frozen; in the cold storage plants it is pumped through coils placed in the rooms which are intended to be kept cool.

211. Sublimation. The pressure of the vapor produced by a solid is usually quite small. For most solids it is practically zero at ordinary temperatures, although our sense of smell often tells us that some vapor is being given off. For ice at 0° the saturated vapor pressure is 4.6 mm of mercury, and decreases rapidly with decrease of temperature.

For some solids the saturated vapor pressure may become quite large at higher temperatures. For iodine it is only 0.01 mm of mercury at 0°, but 47.5 mm at 100°, and 687.2 mm, or almost one atmosphere, at 180°.

As in Art. 202 we may plot a saturated vapor pressure curve representing the conditions under which a solid is in equilibrium with its vapor. We shall call this curve the sublimation curve.

212. The Triple Point. We have seen in Arts. 197, 202, and 211, that under certain conditions of pressure and temperature two states of a substance may be in equilibrium, and that these conditions are represented on a pressure-temperature diagram by curves; namely the fusion curve, the vapor-tension curve and the sublimation curve. In Fig. 107 all three of these curves are drawn for a substance whose melting point decreases with increase of pressure. The curves intersect at a point which is called the triple point and indicates the pressure and temperature at which *all three states are in equilibrium.* For water this point is at the temperature +0°.0076 and at a pressure of 4.6 mm of mercury.

The temperature of a solid body when heated under a pressure higher than that of the triple point will first rise, line *AB* (Fig. 107), until the fusion curve is reached. Then it will melt while the temperature remains constant. After the melting is completed the temperature of the liquid will rise, line *BC*, until the vapor-tension curve is reached; but unless superheated, it cannot remain liquid beyond *C*, which is the boiling point for a given pressure. After all liquid is evaporated, further heating will increase the temperature of the vapor formed, line *CD*.

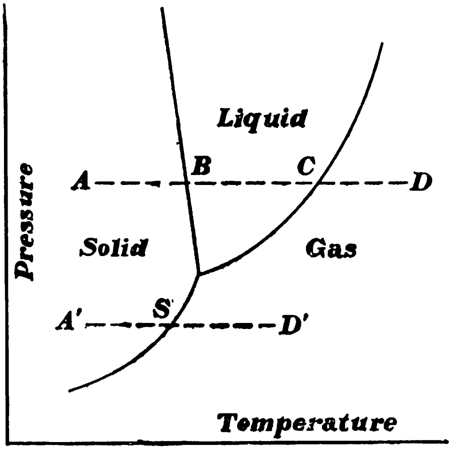

FIG. 107.

If the triple point of a substance lie at a pressure of more than one atmosphere, the solid when heated in an open vessel will not melt, but will sublime, as is shown in the figure by the line *A'SD'*. Camphor and carbon dioxide belong to this class of substances. They have no "normal" melting or boiling point, since under atmospheric pressure the liquid state is impossible for a state of equilibrium.

CHAPTER XXVI

HYGROMETRY

213. The Dew Point. The atmosphere is said to be saturated with moisture if the partial pressure of the water vapor in the air alone equals the vapor tension of water at the temperature of the air; that is, if the water vapor becomes saturated. In this case no further evaporation takes place from a wet surface. In general the atmosphere is not saturated, but if the air be cooled, it may reach a temperature at which it is saturated. Cooling beyond this point results in a condensation of the vapor into water. The dew point is the temperature at which the water vapor present in the air becomes saturated.

The *dew point hygrometer* consists of a vessel with an outer surface of highly polished metal, and containing some volatile liquid, such as ether. By rapid evaporation of this liquid the vessel may be cooled below the dew point. This fact is easily recognized by the formation of a thin film of minute water drops upon the metallic surface. The temperature of the vessel is measured by an accurate thermometer. The vapor tension corresponding to the dew point gives at once the actual vapor pressure in the surrounding air.

Dew does not "fall," but is produced by the condensation of moisture upon surfaces which have been cooled below the dew point. This happens frequently on clear nights when the earth loses much heat by radiation to the sky. Dew forms on the upper surfaces of stones, plants, etc. Moisture is also frequently found collected under cold stones, because the soil underneath remains warm and the air in contact with it has a dew point above the temperature of the stone.

Frost is formed in the same manner as dew, except that the partial pressure of the water vapor is below that of the triple point, so that the vapor will not condense until a temperature below 0° has been reached.

214. Relative Humidity. Relative humidity is the ratio of the pressure of water vapor in the air to the saturated vapor pressure at the temperature of the air. After the dew point has been determined it is only necessary to read from the vapor tension curve the pressure corresponding to the dew point and divide this by the vapor tension corresponding to the temperature of the air. This ratio is the *relative humidity.*

Our sense of dryness and dampness does not depend upon the absolute amount of water vapor in the air. At 0° air is saturated with moisture, if 4.8 g of water are contained in one cubic meter of air, or if the partial pressure is 4.57 mm of mercury. At 0° air containing this amount of water will feel very moist. At 25° the partial pressure of the same amount of water in the air would be (eq. 258) nearly

$$P = \frac{4.57}{273} 298 = 5.0 \text{ mm}$$

of mercury; but the vapor tension at this temperature is 23.52 mm, so that in this case the relative humidity would be only 21.3 per cent, which would make the air appear quite dry. In fact, the capacity of the air for water vapor *nearly doubles for every rise in temperature of 10° C.*

The relative humidity may also be determined by the use of the wet and dry bulb thermometers, or the psychrometer. This instrument consists of two thermometers, the bulb of one being surrounded by a wet piece of muslin, or by a wick dipping into a vessel of water. The drier the air, the more rapidly does the water around the wet bulb evaporate and the lower will be the temperature of this thermometer. The evaporation on the wet bulb may be facilitated by whirling the thermometer through the air (sling thermometer). Tables have been prepared giving direct readings of the relative humidity, from the difference in the readings of the two thermometers and the temperature of the air as determined by the dry bulb thermometer.

215. Condensation of Water in the Atmosphere. We have seen that when air is cooled below the dew point the water vapor begins to condense upon any solid or liquid surface. There are always numerous dust particles in the air which form the nuclei for the water drops. The size of the drops depends upon the number of nuclei present. Very small drops remain suspended in the air for a long time and form mists or fogs. These are frequently formed when moist layers of air are driven by the wind over cold water surfaces, or if they are cooled rather slowly. The dense fogs in large cities are explained by the large number of dust and soot particles in the air.

The condensation accompanying the cooling of moist air may easily be shown by the sudden expansion of moist air under the receiver of an air pump. If a light be viewed through the mist while it forms, beautiful color effects may be observed.

Clouds may be formed in two different ways:

(*a*) The "cumulus" clouds are due to the condensation of water vapor when moist air is carried by an upward air current to regions of lower pressure and its temperature is lowered by adiabatic expansion. These clouds appear in billowy, well-defined shapes, and are due to local disturbances in the atmosphere. They are rarely more than three miles high.

(*b*) The "stratus" clouds are probably due to the mingling of a cold air current with a warmer damp current, either overflowing it or driven up towards it, but moving rather in a horizontal than in an upward direction. The result is an extensive layer, without well-defined shape, covering the sky.

Very small water drops floating at high altitudes in the air produce the optical effect known as a "corona," while a "halo" is a similar effect caused by particles of ice.

The amount of rainfall in different parts of this country varies from 5 to 10 inches per year in the western part of Arizona, Nevada and Utah, to over 80 inches on the northern Pacific coast. The largest rainfall in the Atlantic states is 70 inches per year, in the mountains of North Carolina. The average rainfall in Michigan is between 25 and 35 inches per year.

CHAPTER XXVII

LIQUEFACTION OF GASES

216. Liquefaction by Pressure. We have seen (Art. 202) that condensation of vapor will in general occur at a given temperature when the pressure upon the vapor is raised to a value higher than that which corresponds to this temperature on the vapor tension curve. Vapors are gases, but the term *vapor* is commonly used, if the temperature be not far removed from the point of condensation.

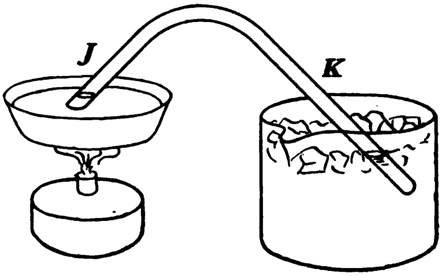

FIG. 108.

Some gases may be easily liquefied at ordinary temperatures by the application of pressure alone, as, for example, chlorine, ammonia and sulphur dioxide. Faraday[1] combined a cooling of the gas with compression, and thus liquefied carbon dioxide.

His apparatus (Fig. 108) consisted of a bent tube, into one end of which, *J*, he had sealed the chemicals for producing the gas, while the other end, *K*, was inserted in a freezing mixture. The gas, when generated, was thus liquefied under its own pressure. Other gases have for a long time withstood all attempts at liquefaction by simple methods.

217. The Critical Point. In 1863 Andrews discovered the fact that carbon dioxide cannot be liquefied by any pressure, however large, if its temperature exceed 31°.1. His experiments[2] may best be described by reference to a diagram (Fig. 109) in which the state of the substance is represented by its volume and the corresponding pressure. Starting with the carbon dioxide gas at a low temperature, say 13°.1, and compressing it isothermally, the pressure constantly increased until it equaled the vapor tension at 13°.1, corresponding to point *B*. At this point liquefaction began. The whole volume of the mixture of liquid *and* gas now decreased while the pressure remained constant, until all the gas was liquefied, point *C*. Upon further compression the pressure of the liquid rapidly increased.

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[1] *Phil. Trans.*, 1823.

[2] *Phil. Trans.*, 1869.

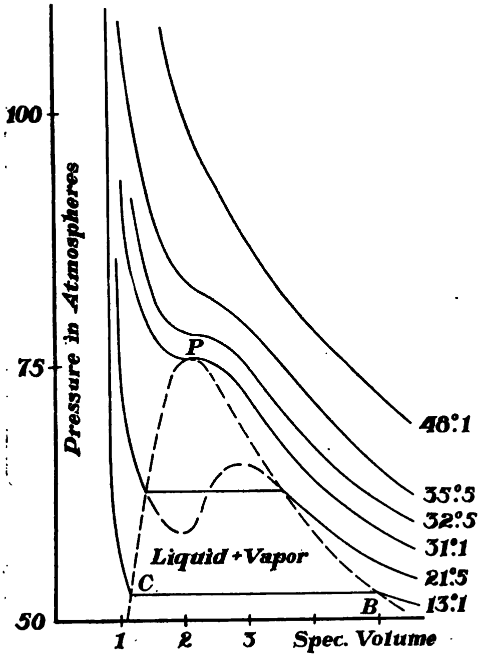

FIG. 109.

Similar curves were obtained at higher temperatures, but it was found that the higher the temperature, the shorter became the horizontal part of the curve *BC*, which represents the mixture of liquid and gas. At 31°.1 no visible liquefaction could be obtained and the curve showed only a distinct bending toward the horizontal before suddenly rising.

At still higher temperatures the curves became smooth and similar to those of a substance obeying the gas law. The area enclosed by the dotted line shows then the conditions under which a mixture of liquid and gas can exist. The temperature of the isothermal on which the substance upon compression ceases to show a free surface, which alone shows a distinction between the liquid and the gaseous state, is called ***the critical temperature.*** The point of contact, *P*, of the region of mixture with this isothermal line is called ***the critical point;*** and the corresponding pressure and specific volume are termed the *critical pressure* and *critical volume* for the substance.

218. Transition through the Critical Point. Let a heavy walled glass tube (Fig. 110), closed at both ends, be filled about half full of ether, the space above the liquid being occupied by ether vapor. The ether is clearly in a state corresponding to some point in the region of mixture. If the tube be slowly heated, it will be found that the liquid expands first slowly, then more rapidly. This is accompanied by a considerable increase of pressure. The meniscus of the liquid becomes flatter, and, as the critical point is reached, it becomes indistinct and finally disappears. The tube is now to all appearances filled with a homogeneous substance, the ether having passed out from the region of mixture. It is idle to discuss the question whether it is now a liquid or a gas. At this state we cannot distinguish between the two.

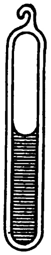

FIG. 110.

When the temperature is allowed to fall again, a hazy cloud forms in the upper part of the tube, the meniscus reappears and the substance is again in two distinct states of aggregation, liquid and gaseous.

TABLE XIV

CRITICAL TEMPERATURE AND PRESSURE

SUBSTANCE	CRIT. TEMP.	CRIT. PRESSURE
Carbon dioxide	31°.1	77 atmos.
Ammonia	131°	113 "
Chlorine	146°	93.5 "
Ether	190°	37 "
Alcohol	240°	64 "
Water	374°	195 "

219. Van der Waals' Equation. It is apparent from the form of the curves (Fig. 109) that the gas law does not accurately represent the state of a gas near the region of liquefaction. In

L. c. 1879 Van der Waals proposed[1] an equation which, while not quite exact, yet represents the actual curves much better than the gas law not only at the right-hand side of the region of mixture, but also on the left-hand side. In the region of mixture this equation gives a curved line as shown by the dotted line (Fig. 109) for the isothermal line corresponding to 21°.5.

Van der Waals' equation is

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \tag{293}$$

in which a and b are constants characteristic of the gas in question provided v represent the specific volume of the gas. This equation, with changed values for a and b, has been found to be applicable to many other gases besides carbon dioxide.

This equation becomes the gas law when the correction terms are small, that is, when the volume is very large in comparison with the volume just before condensation, or when the temperature is very high, in comparison with the critical temperature.

The constant a was introduced in order to take into account the attraction between the molecules of the gas. This attraction will evidently produce a smaller volume, and its effect is equivalent to a pressure which must be added to the external pressure. The attraction between the molecules is proportional to the number of molecules, exerting the attraction, as well as to that of the attracted molecules, or it is proportional to the square of the total number of molecules present. But at constant temperature their number varies directly as the density of the gas and inversely as its volume. The correction factor which must be added to the external pressure is thus equal to $\frac{a}{v^2}$.

The fact that the molecules, however small, still have *some size*, and consequently occupy some small volume, is taken into account by subtracting a small volume, b, from the measured volume. It should be mentioned, however, that b is not the actual volume of the molecules, but represents, as van der Waals has shown, a volume four times as large.

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[1] Translated in *Phys. Mem.*, Phys. Soc. London, vol. 1.

220. The Regenerative Process. Some gases possess a critical temperature below − 100° and must therefore be cooled below this temperature before they can be liquefied. A method which allows a continuous production of these substances in the liquid state was discovered in 1895 by Linde and at about the same time by Hampson.

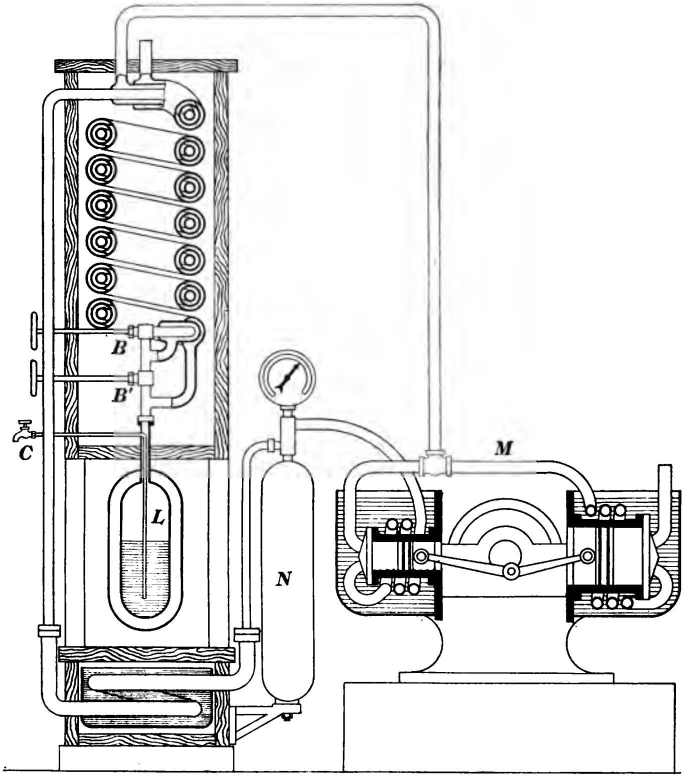

FIG. 111.

The principle used is based upon the fact that gases cool upon expansion (Arts. 181 and 186). The gas is highly compressed to a pressure of 200 atmospheres in a coil immersed in a low temperature bath (Fig. 111), and expands through a needle valve *B*, to a lower pressure, 10 to 20 atmospheres. The cooled gas is then led back through a wider coil completely surrounding the high pressure coil. It consequently cools the compressed gas before this gas reaches the expansion valve. Thus, as the process is continuous, the temperature of the compressed gas is constantly being lowered, until it finally reaches the critical temperature. The gas is then partly liquefied and the liquid collects in the vessel below the valve *B′*, through which the cold gas expands from about 20 atmospheres to atmospheric pressure. The non-liquefied portion of this gas below *B′* is also led back through a wide outer tube surrounding the tube containing the compressed gas and thus assists in cooling this gas still further.

Liquid air machines built on this principle may now be found in many physical laboratories. Hydrogen was first liquefied by Dewar in 1898, and frozen to a foamlike solid by boiling it under reduced pressure.[1] Helium was first liquefied by Kamerlingh-Onnes[2] in 1908. By boiling this liquid under reduced pressure, he attained the temperature – 270°, or ***within 3° of the absolute zero.***

TABLE XV

FREEZING POINT, BOILING POINT AND CRITICAL DATA OF GASES

SUBSTANCE	FREEZING POINT	BOILING POINT	CRIT. TEMP.	CRIT. PRESSURE
Helium	——	– 268°.8	– 268°	2–3 atmos.
Hydrogen . . .	– 260°	– 252°.5	– 240°.8	14 "
Nitrogen . . .	– 210°	– 194°	– 145°	34 "
Oxygen. . . .	– 227°	– 181°	– 118°	50 "
Ethylene . . .	——	– 103°	+ 10°	38 "

Problems

1. Equal masses of water at 25° C and ice at 0° C are mixed. What is the resulting temperature?

2. A mass of 100 g of melting ice is placed in a copper calorimeter whose mass is 100 g, and which contains 500 g of water at 30° C. After the ice is all melted, what will be the temperature of the calorimeter?

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[1] *Chemical News*, 1900.

[2] *Nature*, 1908.

3. A mass of 100 g of ice at $-10°$ C is dropped into a copper calorimeter of mass 100 g, containing 500 g of water. The temperature of the calorimeter is lowered from 30° to 11°.9 C. Determine the heat of fusion of water. Specific heat of ice = 0.5.

4. What is the heat of fusion of water, expressed in British thermal units?

5. How many inches of rain at 10° C must fall in order to melt a sheet of ice $\frac{1}{2}$ in thick?

6. If 10 lb of water should freeze in a cellar containing 3000 cu ft of air, how much would the air be warmed, assuming the specific heat of air as 0.24 and the mass of one cubic foot of air as 0.08 lb?

7. What is the thermal capacity of one cubic foot of air in British thermal units per degree Fahrenheit? in calories per degree Centigrade?

8. A loop of wire 0.02 cm in diameter is placed over a piece of ice, and a 4 kg weight is hung from it. The length of the wire in contact with the ice is 5 cm. Find the average pressure under the wire. At what temperature will the ice under the wire melt?

9. If 5 kg of water at 25° C be placed in a porous jar through which some water can gradually pass and evaporate, how much will have to evaporate in order to cool the remaining water 8° C below the temperature of the surroundings, assuming the water equivalent of the jar to be 50 g?

10. How large a portion of water, undercooled to $-12°$ C will freeze when crystallization takes place? Disregard the water equivalent of the vessel.

11. How much heat is necessary to change 50 g of ice at $-10°$ C to steam at 150° C?

12. An aluminium cup (specific heat 0.21) weighing 80 g contains 483.2 g of water at 18° C. Steam is passed into it until the temperature is raised to 37° C. The calorimeter with water weighs now 579.2 g. Calculate the heat of vaporization.

13. How much would the air in a room $6 \times 5 \times 3$ m be warmed by the condensation of 1 kg of steam in the radiator?

14. Find the correction which must be applied to a thermometer which gives the boiling point of water at 98°.5 C when the barometer stands at 75 cm.

15. At what temperature will water boil in Denver, 5000 ft above sea level?

16. How much water vapor will be produced if 5 kg of water superheated to 105° C suddenly begin to boil under atmospheric pressure? The density of saturated water vapor at 100° C is 0.000606 g/cc.

17. How much water at 100° C and under atmospheric pressure can be vaporized by the burning of 1 kg of coal? Consider all the heat units produced as available for the evaporation of water. See problem 5, p. 231.

18. A mass of 500 g of saturated steam at 100° C is enclosed in a cylinder furnished with a frictionless piston of 400 sq cm area. No heat is supposed to leave the cylinder through the walls, and the vapor is allowed to do work by pushing the piston 10 cm out against a pressure of 75 cm of mercury. How much steam will be condensed?

19. The dew point of air at 25° C is found to be 17° C. Determine from the vapor tension curve on p. 241 the relative humidity. What would be the relative humidity if the temperature of the air had been 20°?

20. How much heat is absorbed when 1 kg of liquid air is boiled under atmospheric pressure and subsequently heated to 20° C? Compare this amount with the heat absorbed by 1 kg of ice melted at 0° C and the water subsequently heated to 20° C. Boiling point of air − 190° C; heat of vaporization of air 50 cal per gram.

21. Assume 2 g of carbon dioxide to be compressed into a space of 8 cm³. Find from Fig. 109 what pressure it exerts at 21°.5 C and what the pressure will be if the temperature be raised through 11° C, the volume remaining the same. How much would the pressure have increased if a perfect gas should have been used instead of carbon dioxide?

22. How much external work is done by 1 kg of water when it freezes at 0° C under atmospheric pressure. What would be the change in the heat of fusion of water if this amount of energy were not included?

23. How much external work is done by the transformation at 100° C of 1 kg of water into steam under normal pressure? How large would be the change in the heat of vaporization of water if this amount of energy were not included? [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

DISTRIBUTION OF HEAT

CHAPTER XXVIII

CONDUCTION

221. Three Modes of Distribution of Heat. In a general way we may speak of the transfer of heat from one body to another, and such transfer is always involved whenever bodies, originally at different temperatures, come to thermal equilibrium (Art. 148). This state is reached, in the case of two bodies thermally insulated from all other bodies, by a mutual approach toward some intermediate temperature. The temperature of one body rises and that of the other falls, by intervals which vary inversely as the thermal capacities of the two bodies in question.

In case one body be connected to some source of heat, the tendency is to maintain this body at some definite temperature and to bring surrounding bodies to the same temperature through the transfer of heat. In all cases, however, heat is transferred *from the body of higher temperature to the body of lower temperature.* From this point of view, difference in temperature is seen to be analogous to difference in level, or to difference in hydrostatic pressure in liquids, and to difference in pressure in connected reservoirs containing gases. In every case the difference in temperature or thermal pressure determines both the direction and the rate of transfer of heat.

Heat may be distributed in any one of three different ways, or, more generally stated, it may be distributed in all three ways at the same time. The three modes of distribution of heat are by *Conduction*, by *Convection* or by *Radiation.*

By *Conduction* is meant the flow of heat through an unequally heated body, or system of bodies, from points of higher to points of lower temperature. This mode of distribution of heat is exemplified in the heating of a metal rod by placing one end in a

Bunsen burner flame. The part in the flame soon becomes quite hot, the molecules of adjacent parts have their motion quickened through the impact of those in the hotter part, and a transfer of heat takes place to points of lower temperature. In this way is set up a steady flow of heat through a rod, between whose ends a definite difference in temperature or thermal difference of level is maintained. Such a rod is said to be a conductor of heat, and the relative ease with which such transfer is made is termed the thermal conductivity of the metal.

By *Convection* we mean the transference of heat by the bodily movement of heated particles of matter. In buildings heated by steam, by hot air or by hot water we find excellent examples of convection of heat.

By ***Radiation*** we mean the transfer of energy from point to point in space by means of waves set up in the ether. The earth is heated by radiation from the sun. The sunlight passing through the window pane brings both light and "heat" into the room. If we hold our hands above a heated stove, the hand is heated both by convection through the air and by radiation. If, however, we hold the hand at the side of the stove and at the same distance from it, the hand will still be heated, but in this case it will be due to ***heating by radiation*** only.

Distribution of heat by radiation is characterized by the absence of any temperature effect upon the medium between the hotter and cooler body. It is true the hotter body loses heat, but from the definition of heat (Art. 146) it is apparent that the energy after it leaves the hotter body can no longer be called heat. It is "energy of radiation" and follows the laws of radiation. But when this form of energy is absorbed by a body, it is retransformed into heat, and the final result will be the gain of heat by one body at the expense of another body at a higher temperature.

From the foregoing it is clear that the so-called radiant "heat" is more closely connected with the subjects of light and electrical waves than with heat. For this reason radiation, including all three phenomena mentioned, will be treated in a later chapter.

222. The Temperature Gradient. It has been shown (Art. 147) how conduction of heat may be explained by the molecular theory of matter as an equalization of molecular kinetic energy. If a rod of metal be placed in a flame, the rise in temperature is at first largely influenced by the specific heat of the substance, since the smaller the specific heat, the more rapidly the temperature will rise.

After a short time, however, the temperature along the bar becomes constant, falling off more or less rapidly from the hotter to the cooler end (Fig. 112), while heat flows steadily through the bar at a definite rate.

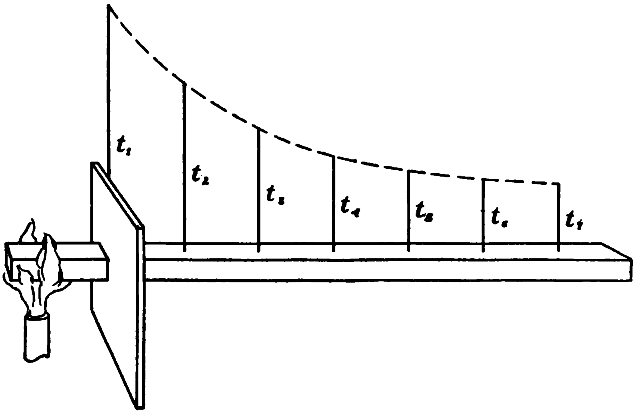

FIG. 112.

The temperature gradient at a point *is the space rate of change of temperature*, or

$$gradient = \frac{t_1 - t_2}{l} \qquad (294)$$

It is represented by the tangent to the temperature curve.

Different bodies vary greatly in their ability to conduct heat. Silver is a very good conductor, iron not so good. A glass rod, placed in a flame, does not become too hot to touch more than a few centimeters from the flame; while we can hold a match until the flame almost touches the fingers.

The better thermal conductor a substance is, the smaller is the temperature gradient, other things being equal, or, the smaller is the decrease of temperature for a given distance from the flame.

We may compare the relative conductivity of copper and iron by taking a copper and an iron wire of the same cross section, twisting their ends together and attaching to them at short distances apart, small pellets of wax. On placing the twisted ends in the flame of a Bunsen burner, the pellets will drop off as soon as the temperature has risen sufficiently to melt the wax. It will be seen that the distance through which the pellets have dropped off is much larger on the copper than on the iron rod. This shows that copper is a better conductor of heat than iron.

223. The Coefficient of Thermal Conductivity. Let two sides of a plate of thickness l be kept at constant temperatures t_1 and t_2, t_1 being larger than t_2. Heat flows through the plate from higher to lower temperature. The quantity of heat passing through any area A is proportional to this area, to the temperature gradient and to the time τ, during which the heat flows. Thus we obtain

$$H = kA\frac{t_1 - t_2}{l}\tau \tag{295}$$

where k is a proportionality factor depending upon the material of the plate. It is called *the coefficient of thermal conductivity* and may be defined as *the time rate of heat conduction per unit area per unit temperature gradient.* It is numerically equal to the heat transferred in unit time through unit area of a plate of unit thickness, if unit difference of temperature be maintained between its two faces.

The numerical value of k depends upon the units chosen for the other quantities. It is said to be expressed in C. G. S. units if H be given in calories, $t_1 - t_2$ in degrees Centigrade and the rest of the quantities in C. G. S. units. The experimental determination of k is very difficult, chiefly on account of the loss of heat from the sides of the body through which the heat passes.

224. Conduction of Heat in Liquids and Gases. In general liquids are poor conductors of heat. Water in a test tube may be boiled in the upper part of the tube, while the lower part contains a piece of ice held down by a piece of wire gauze.

Gases are even poorer conductors than liquids. Porous substances such as wood, wool and asbestos are poor conductors, on account of the large amount of air enclosed in the interstices between the solid material. Such substances are much used to prevent loss of heat from steam pipes, fireless cookers, etc. For this same reason loose clothing is warmer than snugly fitting garments.

In all determinations of the conductivity of fluids it is necessary to avoid currents in the fluid, since they would produce an equilization of temperature by convection instead of by conduction.

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225. The Leidenfrost Phenomenon. If a drop of water be carefully placed by means of a pipette upon a metallic plate, which has been heated to low red heat, the water does not boil, but forms a flattened spheroid, rolls about the surface, evaporating quietly, because a thin film of vapor is formed between the plate and the drop. Owing to the low conductivity of the vapor, heat enters the water but slowly and the drop is at a temperature several degrees below the boiling point. This is called the *Leidenfrost phenomenon*, or the phenomenon of the *spheroidal state.*

If the plate and the drop be connected by an electric circuit containing a bell, the bell does not ring, since there is no contact between plate and drop. When the plate is allowed to cool again, the water makes contact with the plate, boiling sets in and the bell begins to ring.

TABLE XVI

COEFFICIENTS OF THERMAL CONDUCTIVITY IN C. G. S. UNITS

SUBSTANCE	k	SUBSTANCE	k
Silver	1.00	Glass	0.0015
Copper	0.90	Ice	0.005
Zinc	0.25	Wood	0.002
Iron	0.15	Water	0.0015
Mercury	0.016	Air	0.00005

226. Applications. Mention has already been made (Art. 148) of the fact that metals appear colder in cold weather and warmer in warm weather than wood, wool or similar substances. This is easily explained by the difference in thermal conductivity of the substances in question.

A flame does not actually touch a body which is kept at a temperature much lower than that of the flame. Thus water may be boiled in a paper tray over the flame of a Bunsen burner, since the paper remains approximately at the temperature of the boiling water. If we place a piece of wire gauze a few centimeters above a Bunsen burner, turn on and ignite the gas above the gauze, we shall see that the flame will not "strike back" so long as the temperature of the gauze is kept low by conduction of heat away from the flame. If the gauze be lowered over a flame, the flame will not strike through for some time and will do so only after the gauze has become quite hot.

This fact is utilized in Davy's safety lamp. A fine wire gauze entirely surrounds the flame of the lamp. If there be any explosive gases in the mine, they will pass through the gauze into the lamp and ignite there, burning with a bluish flame. For some time the gauze remains cold enough to prevent an explosion of the gases outside. The burning of the gases in the lamp is a warning to the miner to leave the workings until ventilation has restored the atmosphere to a safe condition.

CHAPTER XXIX

CONVECTION

227. Cause of Convection. If a vessel filled with water, in which some solid particles float, be heated, the solid particles show by their motion that a current rises from a point directly over the flame and flows downward again along the cooler walls of the vessel. The upward motion is due to the expansion and decrease of density of the water as the temperature rises. This raises the surface over the heated portions, and the water flows to the lower level above the cooler portions, thus increasing the pressure. The cooler water flows then from points of higher pressure to those of lower pressure and drives the heated portions upward.

In the same manner the density of air is decreased by heating; the light air expands, flows off at the sides, and the cold air presses into the areas of low pressure, causing an upward motion of the heated air. Convection currents in air may often be observed in the "shimmering" of the air over heated plains. The same effect can be shown by passing a beam of light through the air above a heated plate. The heated air is lighter, and has a different index of refraction from that of the cooler air around it, and its movement can be distinctly seen on a screen.

Convection currents do not, strictly, transfer heat, but heated particles. An actual transfer of heat occurs when these particles come into contact with cooler bodies and heat them by conduction.

228. Convection in Liquids. Convection of heat by liquids is employed in the heating of buildings by hot water (Fig. 113). The water rises from the boiler *B* through a system of pipes to the rooms, loses heat in the radiators R_1, R_2 and is led back to the lower part of the boiler by the return pipes. Every hot-water system must be supplied with an open tank, *T*, to allow for the expansion of the water when heated.

Some ocean currents, originating near the equator, have been considered as convection currents. The level of the ocean rises with increase of temperature, and the water flows towards the lower levels farther north, being replaced by cold water from below. Wind, however, is a more important factor in directing the ocean currents, than the extremely small expansions produced by the heating of the ocean.

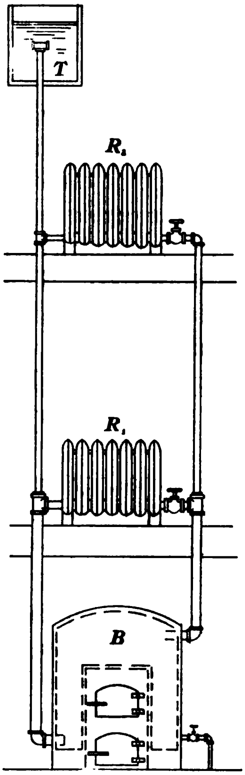

FIG. 113.

229. Convection in Gases. Convection currents of air are in common use for heating buildings and for ventilation. Convection currents are very efficient equalizers of temperatures. The amount of heat carried in this way is quite large. Since one cubic meter of air at 20° C weighs 1.2 kg and its specific heat is 0.24, a cooling of this amount of air to 15° C would release 1440 calories.

The following experiment shows clearly the effect of convection currents of different gases upon a heated body. Inclose a fine straight platinum wire in a glass tube so that it hangs vertically in the axis of the tube. Arrange the experiment so that the tube can be exhausted. Heat the wire to dull red heat by a definite electric current. Upon exhausting the air the wire will lose much less heat than before, owing to the absence of convection currents, and its temperature will be raised to a bright yellow heat. Now let the tube be filled with hydrogen gas. The same electric current as before will not be sufficient to produce a glow in the wire, showing that the convection currents of the hydrogen have a much larger cooling effect than those of air.

The bulbs of incandescent lamps are exhausted and thus loss of heat by convection currents from the glowing filaments to the glass envelope is avoided. The so-called "Dewar flasks" (Fig. 114) are double-walled glass vessels, the space between the walls being an extremely low vacuum, and only a very small heat exchange takes place between the inside and outside of the vessel. Liquids with low boiling points, such as liquid air or liquid hydrogen, may be kept much longer in Dewar flasks than in ordinary vessels since the rate of evaporation is greatly reduced, on account of the slowness with which the heat needed for evaporation passes to the liquid. Recently such flasks have been placed on the market under the name of "Thermos-bottles," and serve equally well either for keeping a liquid hot or for keeping it cold.

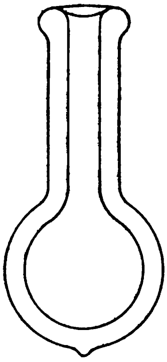

FIG. 114.

230. Convection Currents in the Atmosphere. Land and sea breezes are examples of convection currents. The heating of the land during the day produces an expansion and decrease in pressure of the air, while cooler air from the sea takes the place of the rising air. At sunset the opposite occurs, since the land cools rapidly, while the ocean, owing to the large specific heat of water, remains at the same temperature as during the day.

The trade winds furnish examples of convection currents on a large scale. The heated air in the tropics rises and and cooler air flows in from the north and south. On account of the motion of the earth these cool winds come from an easterly direction. The air which has risen in the tropics flows off towards the poles and descends again to the surface of the earth at a latitude of about 35°. Now since these warm winds descend from greater heights and have therefore a larger velocity toward the east than the surface of the earth, they will flow from the southwest in the northern hemisphere and from the northwest in the southern hemisphere.

Problems

1. Water is boiled in an iron vessel having a heating surface of 400 cm² and a thickness of 4 mm. How much water will be evaporated per hour, if the surface exposed to the fire be kept at 280° C?

2. A lake having a surface area of 9000 square meters is covered by a sheet of ice 5 cm thick. How much heat will pass through the ice in two hours, if the temperature of the air be – 10°, and the ice do not appreciably increase in thickness?

3. The walls of a certain refrigerator have an area of 10,000 cm², are 8 cm thick, and are made of wood. Find how much ice may be expected to melt in a day, if the outside temperature be 25° C?

4. How much coal must be burned to compensate for the loss of heat due to conduction for one day through a glass window 4 mm thick and having an area of 2 square meters, supposing the air in the room next to the glass to be at 25° C, and the outside air at – 10° C? Why is this amount much greater than that actually needed?

5. How much heat would be lost per square decimeter per minute by a man clothed in a fabric 0.3 cm thick, having a coefficient of conductivity equal to 0.00012 C.G.S. units, assuming the temperature of the air to be 5° C, and the temperature of the body 30° C?

6. A balloon of nearly spherical shape and of a capacity of 1000 cubic meters is filled with air of a temperature of 30° C above that of the outside air, which is at 20° C. What is the buoyant force driving the balloon upward?